Annals of Forest Science (2013) 70:743–750
DOI 10.1007/s13595-013-0316-6

ORIGINAL PAPER

Modulus of elasticity declines with decreasing planting density for loblolly pine (*Pinus taeda*) plantations

Ralph L. Amateis · Harold E. Burkhart · Gi Young Jeong

Received: 25 April 2013 / Accepted: 12 July 2013 / Published online: 1 August 2013

Abstract
• ***Context*** Loblolly pine is often grown in intensively managed plantations for wood production. In order to fully evaluate the effects of management practices on wood quality and ultimately value, it is necessary to relate mechanical properties to management practices.
• ***Aims*** The aim of this study was to evaluate the effect of planting density on mechanical properties of lumber recovered from loblolly pine trees from a 27-year-old spacing trial and develop prediction equations for modulus of elasticity and modulus of rupture from stand, tree, and board characteristics.
• ***Methods*** Regression methods were applied to sample trees from three planting densities (2,989, 1,682, and 746 trees ha^{-1}) and used to relate mechanical properties of lumber extracted from the trees to stand, tree, and board characteristics.
• ***Results*** Initial planting density was found to be correlated with modulus of elasticity and, to a lesser extent, with modulus of rupture. Including board characteristics and utilizing the visual grade and board position as regressors produced improved prediction equations.
• ***Conclusions*** The mean modulus of elasticity declines with decreasing planting density while the variability increases, suggesting that planting density is a surrogate for frequency and size of knots. Thus, lower planting densities, while producing more lumber, may produce proportionally fewer boards of greater modulus of elasticity than higher planting densities.

Keywords Wood quality · Spacing · MOE · MOR · Stiffness · Strength

Handling Editor: Jean-Michel Leban

Contribution of the co-authors Amateis: Oversaw data collection, performed statistical analyses, and wrote manuscript.

R. L. Amateis (✉) · H. E. Burkhart
Department of Forest Resources and Environmental Conservation, Virginia Tech, Blacksburg, VA 24061, USA
e-mail: ralph@vt.edu

H. E. Burkhart
e-mail: burkhart@vt.edu

G. Y. Jeong
Department of Wood Science and Engineering, Chonnam National University, 77 Yongbongro Bukgu, Gwangju, South Korea 500-757
e-mail: Gjeong1@jnu.ac.kr

1 Introduction

As demonstrated in the Sudden Sawlog Study (Burton and Shoulders 1974), loblolly pine sawlogs can be produced in fast-grown stands. However, along with this accomplishment comes a concern that the same practices so successful in boosting productivity through intensive silviculture and shorter rotations may have a negative impact on the mechanical properties MOE (modulus of elasticity) and MOR (modulus of rupture) associated with harvested wood (Pearson and Gilmore 1980; Biblis et al. 1993; Biblis et al. 1995; Clark et al. 2008). Thus, in order to fully evaluate effects of management practices on wood quality and ultimately value, it is necessary to relate mechanical properties to management practices.

Pearson and Gilmore (1980) studied mechanical properties of lumber associated with three populations of loblolly pine stands: older natural stands, 25-year-old unmanaged commercial plantations of unimproved trees, and young intensively managed plantations established with genetically improved and fertilized trees. While they found that faster growth rates from plantations were associated with lower values of mechanical properties, they attributed these differences to the younger ages of the plantations rather than differences in growth rates. That is, if allowed to reach ages typical of natural stands, lumber produced from fast-grown plantations would have MOR, MOE, and specific gravity (SG) values similar to those from natural stands. They developed regression equations to predict MOR and MOE from SG, rings per inch and percentage summerwood (SW).

In a study examining the effects of thinning on mechanical properties of loblolly pine lumber, Biblis et al. (2004) found that neither of the properties tested, MOE and MOR, were influenced by thinning. Further, they did not find evidence that variables used to predict MOE and MOR, including SG and percent latewood, were influenced by thinning. Schneider et al. (2008) found similar results for thinned jack pine stands using small clear-wood test samples. They found that thinning did not directly affect prediction models of MOE or MOR, but only indirectly through rings per inch. While both studies found a strong correlation between SG and mechanical properties, Biblis et al. (2004) noted that the strong significance of SG as a predictor of MOE and MOR weakened under the testing protocols of dimensional lumber as compared to small, clear-wood samples. The presence of knots, slope of the grain, and other anomalies found in dimensional lumber can have a strong influence on MOE and MOR that may not be apparent when small, clear-wood samples are tested.

In the absence of thinning, one of the most influential management practices affecting lumber yield and quality is initial planting density. Using lumber recovered from sample trees extracted from a loblolly pine spacing trial, Amateis and Burkhart (2013) found that total lumber production was maximized from larger trees grown at lower planting densities. Even after accounting for the reduction in board quality through the visual grading process and applying prices by grade and dimension, total value of lumber recovered from lower planting densities was significantly greater than from higher densities. Their study provides an initial assessment of value by accounting for quantity and quality off the green chain. However, for a comprehensive assessment of value, it is necessary to account for the effect of planting density on mechanical properties if indeed planting density is found to affect mechanical properties.

The aim of this study was to evaluate the effect of planting density on mechanical properties of lumber recovered from loblolly pine trees from a 27-year-old spacing trial and develop prediction equations for MOE (modulus of elasticity) and MOR (modulus of rupture) from stand, tree, and board characteristics.

2 Materials and methods

2.1 Study and treatment plots

The Forest Modeling Research Cooperative established a set of loblolly pine (*Pinus taeda* L.) spacing trials in the early spring, 1983. At each of the four locations (two in the Piedmont of Virginia and one each in the Coastal Plain areas of Virginia and North Carolina), three replicates containing 16 treatment plots were established. Only the two Coastal Plain sites each with three plots of the square planting densities of 1.8 m×1.8 m (2,989 trees ha^{-1}), 2.4 m×2.4 m (1,682 trees ha^{-1}), and 3.66 m×3.66 m (746 trees ha^{-1}) were considered. Additional information about the trial design and a comprehensive summary of research results over the life of the study have been published (Amateis and Burkhart 2012). The quantity, quality, and value of lumber off the green chain recovered from sample trees at age 27 can be found in Amateis and Burkhart (2013).

Age 27 data were collected on all live standing trees from the 2,989, 1,682, and 746 trees ha^{-1} treatment plots. Standing tree measurements relevant to these analyses included diameter at breast height, total height, and product categorization as either sawtimber or pulpwood. To be categorized as sawtimber, a tree had to have a dbh of at least 22 cm with a 5 m butt log free of damage and disease, and be straight such that a line connecting the center of the stem at any two points above the stump would not lie outside the tree bole (Amateis and Burkhart 2013).

2.2 Sample tree selection

From each replicate at each location, up to six trees were selected at random from the pool of qualifying sawtimber quality trees on each of the treatment plots. For plots that did not have six qualifying trees in a particular replicate, additional qualifying trees from the same treatment plot in the other two replicates at the same location were used. An attempt was made to obtain 18 sample trees representing each treatment plot at each of the two locations for a total of 108 sample trees. Due to the lack of suitable sample trees from the 2,989 trees ha^{-1} plots, however, the total number of sample trees representing that treatment was 25. Thus a total of 97 sample trees was obtained.

Sample trees were felled and total height and height to live crown were obtained by tape. The butt log was cut from the tree and log length and large and small end diameters were measured. Tags identifying each log by location, replication, plot, and tree were attached to the ends of sample logs which were then skidded off the plots and subsequently transported to the sawmill.

2.3 Milling

A portable Wood-Mizer sawmill was used to recover lumber from the sample logs. All logs were milled employing the same operator and sawn into structural 5-cm and non-structural 2.5-cm boards according to mill-run specifications. The goal was to maximize the lumber yield recovered from each log. Widths were 10, 15, or 20 cm; lengths varied from 2.5 to 5 m.

Off the carriage, each piece of fresh lumber was identified by location, replicate, treatment plot, tree (log), board position within the log (inside or outside) and assigned a board number. The length, width, and thickness dimensions were recorded. All 5-cm-thick boards were visually graded green as #1, #2, or #3 according to national grading rules for structural light framing lumber (National Grading Rule Committee 2004) by a certified lumber grader. Following green grading, all #1 and #2 boards were racked on drying sticks and air-dried for several months reaching a stable moisture content of about 20 % (Table 1).

Out of the 5-cm-thick boards green graded #1 and #2, one inside board (containing pith) and one randomly selected outside board per log were chosen for testing of mechanical properties resulting in 184 test boards (ten trees had only one suitable board for testing). The center 2 m of each board was extracted. If the sample board had a width of 15 or 20 cm, one edge was randomly selected and set down on the carriage. The board was then ripped to a width of 10 cm. Thus, each sample board was of dimension 5 cm thick × 10 cm wide × 2 m long.

The test boards were placed into an Ebac LD3000 dehumidifier kiln and dried to 10 % moisture content. Following drying, each board was re-graded by the same certified grader. Of the 184 #2 and better green graded boards, the drying process resulted in 54 boards being down-graded to #3. All test boards were then shipped to the Timber Products Inspection (TPI) lab. Upon receipt at TPI, the boards were stored at ambient conditions of 24°C and 50 % relative humidity until testing.

In the lab, each board was subjected to edge-wise bending in accordance with ASTM D4761 testing protocols using a Tinius Olsen machine. A static bending load was applied to each board across a span of 173 cm (span to depth ratio of 17) at a testing speed that applied stress at a rate of 0.02758 GPa min^{-1}. Deflection data were collected until the load reached 138 kN whereupon each board was loaded until failure. Data collected included strength (MOR), stiffness (MOE), and cause of failure. Following testing, a 2.5-cm section was cut from each board as close to the point of failure as possible and used to determine moisture content (MC) at time of testing and SG in accordance with ASTM D4442 and D2395, respectively. In addition, rings per 2.5-cm increment (RPI) and percent summerwood (SW) were obtained. One board with a very low MOE (2.8 GPa) was considered an outlier and removed from the dataset, leaving a total sample size of 183. Table 2 summarizes the data collected from the testing procedure. Figures 1 and 2 present the observed distributions of MOE and MOR, respectively, for the three planting densities. Table 3 shows the cumulative proportion of boards grouped by MOE class and visual grade class for each treatment plot. Table 4 presents, in a similar format, the MOR data. The fifth percentile values of MOE for the 746, 1,682, and 2,989 trees ha^{-1} planting densities were 3.962, 4.319, and 6.540 GPa, respectively. The fifth percentile values of MOR for the 746, 1,682, and 2,989 trees ha^{-1} planting densities were 0.0183, 0.0215, and 0.0274 GPa, respectively.

Table 1 Tally of 263 5-cm thick boards sawn from sample trees by planting density, visual grade green, and board width

Density[a]	Visual grade green #2 and better			Visual grade green #3		
	10 cm	15 cm	20 cm	10 cm	15 cm	20 cm
2,989	36	15		3	1	
1,682	33	50	2	2	5	
746	20	74	12		9	1

[a] Planting density (trees per hectare)

2.4 Regressions

Regression methods were used to develop models for predicting mechanical properties MOE and MOR of 5-cm kiln-dried lumber. Stand, tree, and board variables available for use as possible regressor variables included planting density (PD), dbh (D), tree height (H), crown length (CL), large end log diameter (Dl), small end log diameter (Ds), and log taper (Ds/Dl). Board variables MOE, MOR, SG, rings per 2.5-cm increment (RPI) and percent summerwood (SW) were averaged by tree resulting in a dataset of 94 observations containing one stand variable (PD) and tree and mean board variables for each tree (mean board variables for each tree denoted by $\overline{\mathrm{MOE}}$, $\overline{\mathrm{MOR}}$, $\overline{\mathrm{SG}}$, $\overline{\mathrm{RPI}}$, $\overline{\mathrm{SW}}$). Stand age and site index were not useful here because all trees were of the same age and site index varied little among treatment plots and also between the two coastal plain locations. Table 5 presents Pearson correlation coefficients between all variables and $\overline{\mathrm{MOE}}$ and $\overline{\mathrm{MOR}}$.

The first objective was to examine correlations between $\overline{\mathrm{MOE}}$ and $\overline{\mathrm{MOR}}$ values and associated stand and tree variables readily available as inventory data to determine which might be significant predictor variables in a regression equation:

$$\overline{\mathrm{MOE}} = f\left(\mathrm{PD}, \mathrm{D}, \mathrm{H}, \mathrm{CL}, \mathrm{Dl}, \mathrm{Ds}, \left(\frac{\mathrm{Ds}}{\mathrm{Dl}}\right)\right) + \varepsilon \qquad (1)$$

$$\overline{\mathrm{MOR}} = f\left(\mathrm{PD}, \mathrm{D}, \mathrm{H}, \mathrm{CL}, \mathrm{Dl}, \mathrm{Ds}, \left(\frac{\mathrm{Ds}}{\mathrm{Dl}}\right), \overline{\mathrm{MOE}}\right) + \varepsilon \qquad (2)$$

where ε is an error term and other variables are as defined previously. Applying methods similar to Lei et al. (2005) and Liu et al. (2007), stepwise regression procedures were used to relate the regressor variables of Eqs. (1) and (2) to $\overline{\mathrm{MOE}}$ and $\overline{\mathrm{MOR}}$. Variables significant at α=0.05 remained in the regression.

A second set of more comprehensive equations that included mean board characteristics for each tree $\overline{\mathrm{SG}}$, $\overline{\mathrm{RPI}}$, and $\overline{\mathrm{SW}}$

Table 2 Summary of the wood characteristics for 183 kiln-dried sample boards by planting density and board grade sawn from butt logs from sample trees from the loblolly pine spacing trials

			Mean (minimum, maximum in parentheses)				
Planting density (trees ha^{-1})	Visual board grade[a]	Number sample boards	Rings 2.5 cm^{-1}	Summer wood (%)	SG (od vol^{-1} at 10 % MC)	MOE (GPa)	MOR (GPa)
2,989	2	40	6.1 (3, 9)	29.2 (12, 48)	0.47 (0.38, 0.60)	9.48 (6.41, 14.5)	0.049 (0.024, 0.085)
	3	6	5.5 (5, 6)	28.0 (20, 38)	0.47 (0.45, 0.50)	7.08 (4.43, 8.40)	0.048 (0.041, 0.064)
	All	46	6.0 (3, 9)	29.0 (12, 48)	0.47 (0.38, 0.60)	9.17 (4.43, 14.5)	0.049 (0.024, 0.085)
1, 682	2	47	5.1 (2, 8)	28.6 (14, 50)	0.46 (0.37, 0.59)	8.80 (4.84, 13.2)	0.047 (0.021, 0.091)
	3	20	4.8 (3, 8)	28.1 (12, 43)	0.44 (0.39, 0.51)	5.69 (3.34, 7.94)	0.032 (0.014, 0.059)
	All	67	5.0 (2, 8)	28.4 (12, 50)	0.46 (0.37, 0.59)	7.87 (3.34, 13.2)	0.042 (0.014, 0.091)
746	2	42	4.4 (2, 7)	26.0 (10, 50)	0.47 (0.40, 0.56)	8.38 (4.25, 15.4)	0.050 (0.020, 0.104)
	3	28	3.9 (2, 8)	28.5 (15, 56)	0.43 (0.38, 0.53)	5.36 (3.22, 8.24)	0.031 (0.016, 0.057)
	All	70	4.2 (2, 8)	27.0 (10, 56)	0.46 (0.38, 0.56)	7.17 (3.22, 15.4)	0.042 (0.016, 0.104)

[a] 2=#2 or better; 3=#3

were also considered:

$$\overline{\text{MOE}} = f\left(\text{PD, D, H, CL, Dl, Ds}, \left(\frac{\text{Ds}}{\text{Dl}}\right), \overline{\text{SG}}, \overline{\text{RPI}}, \overline{\text{SW}}\right) + \varepsilon \quad (3)$$

$$\overline{\text{MOR}} = f\left(\text{PD, D, H, CL, Dl, Ds}, \left(\frac{\text{Ds}}{\text{Dl}}\right), \overline{\text{SG}}, \overline{\text{RPI}}, \overline{\text{SW}}, \overline{\text{MOE}}\right) + \varepsilon \quad (4)$$

From the stepwise procedures a set of OLS regression equations was identified for predicting $\overline{\text{MOE}}$ and $\overline{\text{MOR}}$ from only stand and tree characteristics [Eqs. (1a) and (2a)] and another set for predicting $\overline{\text{MOE}}$ and $\overline{\text{MOR}}$ from stand, tree, and mean board characteristics utilizing all boards [Eqs. (3a) and (4a)]:

$$\overline{\text{MOE}} = a_0 + a_1(\text{PD}) + a_2\text{H} + \varepsilon_1 \quad (1a)$$

$$\overline{\text{MOR}} = b_0 + b_1\overline{\text{MOE}} + \varepsilon_2 \quad (2a)$$

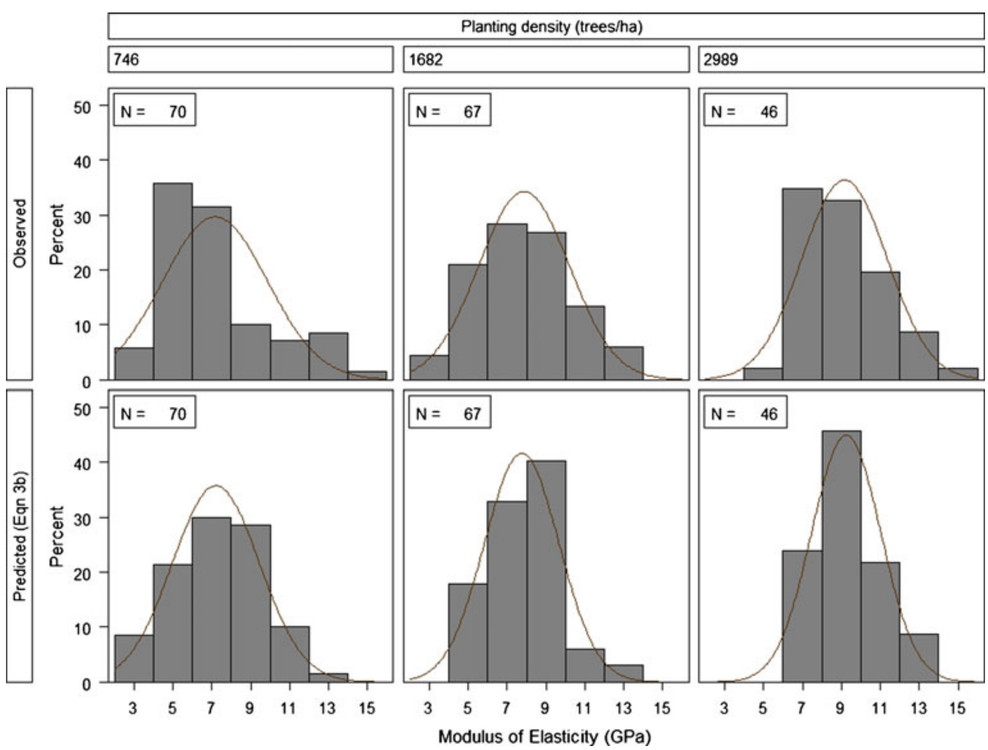
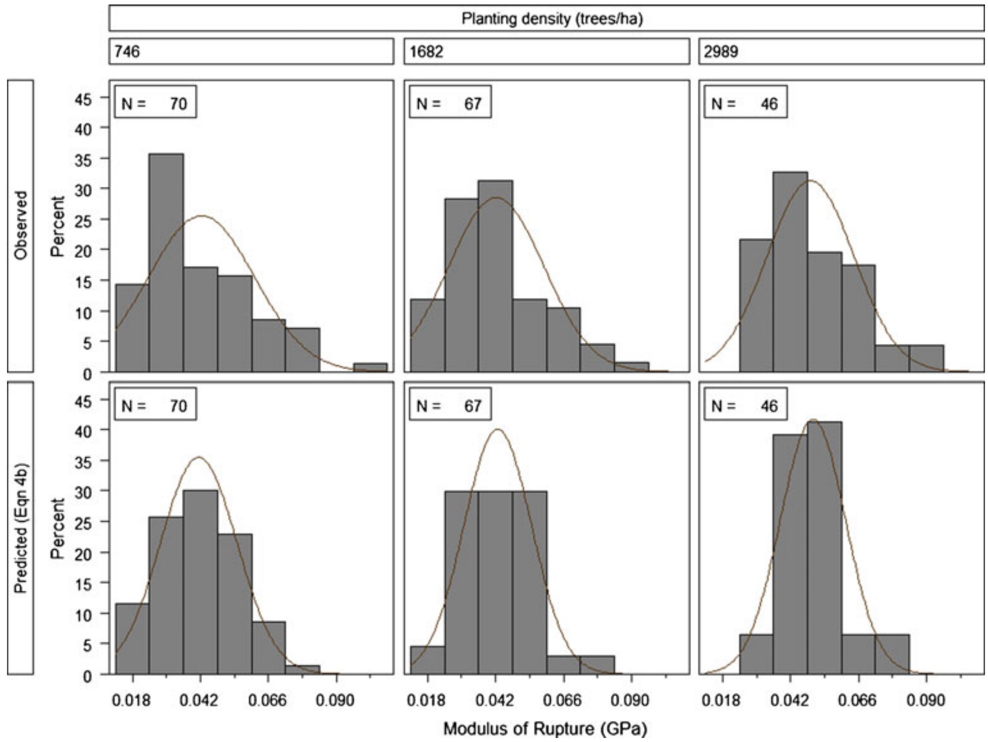


Fig 1 Comparison of the observed and predicted [using Eq. (3b)] distributions of MOE values and the fitted normal distribution by planting density for 183 test boards.

Fig 2 Comparison of the observed and predicted [using Eq. (4b)] distributions of MOR values and the fitted normal distribution by planting density for 183 test boards



$$\overline{\mathrm{MOE}} = c_0 + c_1\mathrm{PD} + c_2\mathrm{D} + c_3\overline{\mathrm{RPI}} + c_4\overline{\mathrm{SG}} + \varepsilon_3 \tag{3a}$$

$$\overline{\mathrm{MOR}} = d_0 + d_1\overline{\mathrm{MOE}} + d_2\overline{\mathrm{SG}} + \varepsilon_4 \tag{4a}$$

Table 3 Cumulative proportions for 183 kiln-dried sample boards by planting density, visual grade, and MOE class

	Planting density (trees ha^{-1})					
	2,989		1,682		746	
MOE (GPa)	#2[a] (40)	#3[b] (6)	#2[a] (47)	#3[b] (20)	#2[a] (42)	#3[b] (28)
<4.999		0.167	0.021	0.200	0.071	0.357
5.0–5.999			0.106	0.600	0.190	0.750
6.0–6.999	0.10	0.500	0.191	0.800	0.429	0.964
7.0–7.999	0.35		0.340	1.000	0.571	
8.0–8.999	0.475	1.000	0.511		0.595	1.000
9.0–9.999	0.625		0.723		0.714	
10.0–10.999	0.725		0.894		0.786	
11.0–11.999	0.875		0.915		0.833	
12.0–12.999	0.925		0.979		0.952	
>13.0	1.000		1.000		1.000	

[a] Visual grade #2 and better (total number of sample boards in parentheses)

[b] Visual grade #3 (total number of sample boards in parentheses)

where $a_0 - d_2$ are parameter estimates and $\varepsilon_1 - \varepsilon_4$ are error terms for Eqs. (1a)–(4a), respectively. Thus, Eqs. (1a)–(4a) have 97 observations.

To examine the question of how useful visual grade and board position (inside or outside) might be for predicting mechanical properties, Eqs. (1a)–(4a) were expanded by allowing visual grade and position to enter the regressions as a predictor variables at α=0.05:

$$\mathrm{MOE} = a_0 + a_1(\mathrm{PD}) + a_2\mathrm{H} + a_3\mathrm{VG2} + \varepsilon_1 \tag{1b}$$

$$\mathrm{MOR} = b_0 + b_1\mathrm{MOE} + b_2\mathrm{BP} + \varepsilon_2 \tag{2b}$$

$$\mathrm{MOE} = c_0 + c_1\mathrm{PD} + c_2\mathrm{D} + c_3\mathrm{RPI} + c_4\mathrm{SG} + c_5\mathrm{VG2} + \varepsilon_3 \tag{3b}$$

$$\mathrm{MOR} = d_0 + d_1\mathrm{MOE} + d_2\mathrm{SG} + d_3\mathrm{BP} + \varepsilon_4 \tag{4b}$$

where MOE, MOR, SG, and RPI in Eqs. (1b)–(4b) are as defined previously and VG2=1 if visual grade 2 or better; 0 otherwise, and BP=1 if board position is inner (contains pith); 0 otherwise.

3 Results

The OLS fit of Eq. (1a) produced a prediction equation for $\overline{\mathrm{MOE}}$ from regressors PD and H with R^2 of 0.26. Residual

Table 4 Cumulative proportions for 183 kiln-dried sample boards by planting density, visual grade, and MOR class

	Planting density (trees ha^{-1})					
	2,969		1,682		746	
MOR (GPa)	#2[a] (40)	#3[b] (6)	#2[a] (47)	#3[b] (20)	#2[a] (42)	#3[b] (28)
<0.020				0.100	0.024	0.143
0.020–0.0299	0.100		0.170	0.450	0.119	0.536
0.030–0.0399	0.350		0.426	0.750	0.381	0.750
0.040–0.0499	0.550	0.667	0.617	0.950	0.548	1.000
0.050–0.0599	0.725	0.833	0.766	1.000	0.714	
0.060–0.0699	0.850	1.000	0.872		0.810	
0.070–0.0799	0.950		0.979		0.952	
0.080–0.0899	1.000				0.976	
0.090–0.0999			1.000			
>0.100					1.000	

[a] Visual grade dry #2 and better (total number of sample boards in parentheses)

[b] Visual grade dry #3 (total number of sample boards in parentheses)

plots across PD, H, and other tree variables did not show trends that would indicate bias. The stepwise procedure on Eq. (2) resulted in prediction Eq. (2a) which uses only $\overline{\mathrm{MOE}}$ as a significant predictor of $\overline{\mathrm{MOR}}$. The R^2 for Eq. (2a) was 0.57 which reflects only the high correlation between $\overline{\mathrm{MOE}}$ and $\overline{\mathrm{MOR}}$.

Equation (3a) included mean board characteristics in combination with stand and tree variables as predictors of $\overline{\mathrm{MOE}}$ and produced a significant regression of $\overline{\mathrm{MOE}}$ from PD, D, $\overline{\mathrm{RPI}}$, and $\overline{\mathrm{SG}}$ with a R^2 of 0.61. A significant regression for predicting $\overline{\mathrm{MOR}}$ from $\overline{\mathrm{MOE}}$ and $\overline{\mathrm{SG}}$ [Eq. (4a)] was obtained with a R^2 of 0.66.

Since previous work has indicated that a curvilinear relationship may exist between MOE and certain board characteristics for some species (Zhang 1994; Zhang 1997), an effort was made to improve Eqs. (1a)–(4a) by expressing the regressors as power functions and structuring the equations as nonlinear regressions, then refitting using nonlinear regression methods. The nonlinear model fits were not an improvement over the linear OLS fits.

Because cross-equation correlations exist among variables used to predict $\overline{\mathrm{MOE}}$ and $\overline{\mathrm{MOR}}$ for the system of Eqs. (1a) and (2a), and the more comprehensive system (3a) and (4a), some gain in modeling efficiency may be achieved by using joint-generalized least squares, also termed seemingly unrelated regression (SUR). For plantation-grown black spruce, Lei et al. (2005) found that using SUR produced a reduction in standard errors of the parameter estimates ranging from 1–15 % over the OLS estimates. Therefore, the two systems of equations for loblolly pine were fitted using SUR methods and results compared to the OLS fits. Only a very small gain in efficiency as expressed by a reduction in the standard error of the parameter estimates was achieved using SUR on these data. For the system of Eqs. (1a) and (2a), the standard error of each of the five parameters was reduced by less than 0.4 %. For the system of Eqs. (3a) and (4a), SUR produced a reduction of less than 0.1 % in the standard errors of only three of the eight parameters.

Predicting MOE at the board level and utilizing the visual grade as a regressor produced prediction Eqs. (1b) and (3b) with R^2 values of 0.40 and 0.70, respectively. The variable VG2 was highly significant for predicting MOE in Eqs. (1b) and (3b), but not significant for predicting MOR in (2b) and (4b). For predicting MOR at the board level, board position

Table 5 Pearson correlation coefficients between $\overline{\mathrm{MOE}}$, $\overline{\mathrm{MOR}}$, and potential stand, tree, and mean board regressor variables for 94 sample trees

	Stand	Tree					Board				
Variable[a]	PD	D	H	H/D	CL	Ds/Dl	$\overline{\mathrm{RPI}}$	$\overline{\mathrm{SG}}$	$\overline{\mathrm{SW}}$	$\overline{\mathrm{MOE}}$	$\overline{\mathrm{MOR}}$
$\overline{\mathrm{MOE}}$	0.42	−0.15	0.05	0.24	−0.02	−0.02	0.59	0.68	0.40	1.00	0.77
$\overline{\mathrm{MOR}}$	0.23	−0.08	0.02	0.13	0.09	−0.09	0.50	0.74	0.37	0.77	1.00

[a] $\overline{\mathrm{MOE}}$ =mean MOE (in gigapascal) per tree, $\overline{\mathrm{MOR}}$ =mean MOR (in gigapascal) per tree, PD=planting density (trees per hectare), D=tree dbh (in meter), H=tree height (in meter), H/D=height/dbh ratio, CL=crown length (in meter), Ds/Dl=ratio of log small end diameter to large end diameter, $\overline{\mathrm{RPI}}$ =mean rings per 2.5-cm increment, $\overline{\mathrm{SG}}$ =mean specific gravity, $\overline{\mathrm{SW}}$ =mean percent summerwood.

was a significant regressor. R^2 values of 0.61 and 0.65 were obtained for prediction Eqs. (2b) and (4b), respectively. Applying SUR methods to estimate parameters for these board-level models produced only minimal reduction of the standard errors of the parameter estimates. Table 6 presents final parameter estimates for Eqs. (1a)–(4b) using SUR methods. Figures 1 and 2 present predicted distributions for MOE and MOR using Eqs. (3b) and (4b), respectively.

4 Discussion

Overall, the results of this study are similar to previous work relating mechanical properties of lumber obtained from static bending tests to stand, tree, and board characteristics. For stands of black spruce, Lei et al. (2005) and Liu et al. (2007) found stand density to be significantly and positively correlated with MOE as was found in this loblolly pine plantation study. Similarly, Zhang et al. (2006) working with jack pine and Moore et al. (2009) with Sitka spruce found similar significant positive correlations between stand density and lumber stiffness following early thinnings. Apparently, stand density is a reasonable surrogate for branchiness which affects MOE through the size and frequency of knots. The prediction equation in this loblolly pine study based only on planting density and tree height has relatively low precision (R^2=0.26) compared with the equations fitted for 55- to 100-year-old black spruce trees by Lei et al. (2005) and Liu et al. (2007) who reported R^2 values in the 0.55–0.65 range for mean MOE. The 0.26 R^2 value for the loblolly pine data is comparatively low which reflects the inherent variability of the MOE and MOR data in the sample (Tables 3 and 4) and may also be related to the sample size and tree age.

Mechanical properties were found to be more highly correlated to board characteristics than tree and stand characteristics. Equations including mean board characteristics $\overline{\mathrm{RPI}}$ and $\overline{\mathrm{SG}}$ increased model precision considerably for predicting $\overline{\mathrm{MOE}}$ at the tree level over equations based on stand and tree characteristics alone. As expected, $\overline{\mathrm{MOR}}$ was strongly correlated with $\overline{\mathrm{MOE}}$.

Testing full-sized dimension lumber as in this study resulted in high variability of the mechanical properties. Biblis et al. (2004) noted that the strong correlation of SG of small clear test specimens to mechanical properties weakens when applied to full-sized dimension lumber. It is likely the effects of knots, slope of grain, and other abnormalities found in dimension lumber increased the variability of MOE and MOR within the sample and hence reduced the precision of resultant prediction equations. Including the visual grade category in the regression equations increased the R^2 for the mean $\overline{\mathrm{MOE}}$ equation from 0.26 [Eq. (1a)] to 0.40 [Eq. (1b)] and for the individual board MOE equation from 0.61 [Eq. (3a)] to 0.70 [Eq. (3b)] (Table 6). Clearly, visual grade is a useful indicator of the effect of knot size, frequency, and position on MOE.

The MOE values of the sample boards (Table 3) show considerable variability for the #2 and better boards for all planting densities. The mean MOE declines with decreasing planting density while the variability increases, again suggesting that planting density is a surrogate for frequency and size of knots. Thus, lower planting densities, while producing more lumber (Amateis and Burkhart 2012), may produce proportionally fewer boards of greater MOE than higher planting densities. For visual grade #3, both the mean and variation of MOE appear to be less sensitive to changes in planting density than grade #2 or better. These trends can

Table 6 Equations (1a)–(4b) with parameter estimates and fit statistics for predicting mechanical properties from planting density, tree, and board characteristics using SUR methods

Equation*		MSE	R^2
Tree-level prediction equations			
$\overline{\mathrm{MOE}} = -4.9296 + 0.00135(\mathrm{PD}) + 0.1475\mathrm{H}$	(1a)	3.19	0.26
$\overline{\mathrm{MOR}} = 0.007944 + 0.004558\overline{\mathrm{MOE}}$	(2a)	0.000076	0.57
$\overline{\mathrm{MOE}} = -10.8372 + 0.000695\mathrm{PD} + 10.337\mathrm{D} + 0.44825\overline{\mathrm{RPI}} + 27.314\overline{\mathrm{SG}}$	(3a)	1.725	0.61
$\overline{\mathrm{MOR}} = -0.04688 + 0.00307\overline{\mathrm{MOE}} + 0.1449\overline{\mathrm{SG}}$	(4a)	0.000061	0.66
Board-level prediction equations			
MOE=−6.099+0.000914(PD)+0.14478H+2.95587VG2	(1b)	3.941	0.40
MOR=0.00988+0.00467MOE−0.00516BP	(2b)	0.000118	0.61
MOE=−11.7733+0.000435(PD)+9.2976D+0.3867RPI+28.370SG+2.0709VG2	(3b)	1.9854	0.70
MOR=−0.03401+0.00356MOE+0.1142*SG*−0.00475BP	(4b)	0.000108	0.65

*$\overline{\mathrm{MOE}}$ =mean MOE (in gigapascal) of boards per tree, $\overline{\mathrm{MOR}}$ =mean MOR (in gigapascal) of boards per tree, MOE=modulus of elasticity (in gigapascal), MOR=modulus of rupture (in gigapascal), PD=planting density (trees per hectare), H=total height (in meter), D=diameter breast height (in meter), $\overline{\mathrm{RPI}}$ =mean rings per 2.5-cm increment, $\overline{\mathrm{SG}}$ =mean specific gravity, RPI=rings per 2.54-cm increment, SG=specific gravity, VG2=1 if visual grade is #2 or better; 0 otherwise, BP=1 if board position is inner (containing pith); 0 otherwise.

impact end use and hence economic value of recovered lumber when particular design values are required.

The equations presented here can be used for a variety of purposes to assist practitioners with assessing the value of loblolly pine plantations. For alternative planting densities, $\overline{\text{MOE}}$ for lumber obtained from standing trees that meet sawtimber specifications can be estimated using Eq. (1a) and, when visual grades of sample lumber are available, Eq. (1b). Distributions of $\overline{\text{MOE}}$ by visual grade across height and/or diameter class can be useful for estimating mechanical properties of the product mix and ultimately value following harvest. Stand and stock table output from growth and yield models can be augmented with estimates of MOE and MOR for particular stand and tree characteristics. Where machine stress rating of visually graded boards for specific applications is not available, the equations presented here can be used to predict MOE and MOR.

The data used in this study are limited to planting densities ranging from 2,989 to 746 trees ha^{-1}. They should be indicative of stands established with first generation seed orchard seedlings on coastal plain sites similar to those included in the spacing trial. Other than early competition control, no intermediate treatments were imposed prior to harvest at age 27 years. Changes in any of these factors can influence the mechanical properties of lumber harvested from loblolly pine plantations. Still, the results of this study suggest that decisions concerning planting density made at time of stand establishment will affect the mechanical properties of lumber obtained following harvest.

Acknowledgments Financial support from the Forest Modeling Research Cooperative at Virginia Tech and the Acorn Alcinda Foundation is gratefully acknowledged.

References

Amateis RL, Burkhart HE (2012) Rotation-age results from a loblolly pine spacing trial. South J Appl For 36:11–18

Amateis RL, Burkhart HE (2013) Relating quantity, quality and value of lumber to planting density for loblolly pine plantations. South J Appl For 37:97–101

Biblis EJ, Brinker R, Carino HF, McKee CW (1993) Effect of stand age on structural properties and grade compliance of lumber from loblolly pine plantation timber. For Prod J 43:23–28

Biblis EJ, Carino H, Brinker R, McKee CW (1995) Effect of stand density on flexural properties of lumber from two 35-year-old loblolly pine plantations. Wood and Fiber Sci 27:25–33

Biblis EJ, Meldahl R, Pitt D, Carino HF (2004) Predicting flexural properties of dimension lumber from 40-year-old loblolly pine plantation stands. For Prod J 54:109–113

Burton JD, Shoulders E (1974) Fast-grown, dense, loblolly pine sawlogs: a reality. J For 72:637–641

Clark A III, Jordan L, Schimleck L, Daniels RF (2008) Effect of initial planting spacing on wood properties of unthinned loblolly pine at age 21. For Prod J 58:78–83

Lei YC, Zhang SY, Jiang Z (2005) Models for predicting lumber bending MOR and MOE based on tree and stand characteristics in black spruce. Wood Sci Technol 39:37–47

Liu C, Zhang SY, Cloutier A, Rycabel T (2007) Modeling lumber bending stiffness and strength in natural black spruce stands using stand and tree characteristics. For Ecol Manag 242:648–655

Moore J, Achim A, Lyon A, Mochan S, Gardiner B (2009) Effects of early re-spacing on the physical and mechanical properties of Sitka spruce structural timber. For Ecol Manag 258:1174–1180

Pearson RG, Gilmore RC (1980) Effect of fast growth rate on the mechanical properties of loblolly pine. For Prod J 30:47–54

Schneider R, Zhang SY, Swift DE, Bégin J, Lussier JM (2008) Predicting selected wood properties of jack pine following commercial thinning. Can J For Res 38:2030–2043

Zhang SY (1994) Mechanical properties in relation to specific gravity in 342 Chinese woods. Wood and Fiber Sci 26:512–526

Zhang SY (1997) Wood specific gravity-mechanical property relationship at species level. Wood Sci Technol 31:181–191

Zhang SY, Chauret G, Swift DE, Duchesne I (2006) Effects of precommercial thinning on tree growth and lumber quality in a jack pine stand in New Brunswick, Canada. Can J For Res 36:945–952