

RESEARCH PAPER



Open Access



A growth-effective age-based periodic site-index for the estimation of dynamic forest site productivity under environmental changes

Chaofang Yue^{1*}, Hans-Peter Kahle², Joachim Klädtke¹ and Ulrich Kohnle¹

Abstract

Key message A novel periodic site index is introduced for the quantification of dynamic forest site productivity. The measure is age-independent, sensitive to environmental changes and efficient for the estimation and prediction of stand height and stand volume increment.

Context Accurate and up-to-date prediction of site productivity is crucial for the sustainable management of forest ecosystems, especially under environmental changes.

Aims The aim of this study was to introduce a novel concept: a periodic site index based on growth-effective age for the quantification of dynamic forest site productivity.

Methods The growth-effective age based periodic site index is estimated from repeated or multi-temporal measurements of stand dominant height. Furthermore, a recursive procedure to update the underlying site index model is presented by using repeated measurements of stand dominant height. The database used in this study comprised repeated measurements of 945 Norway spruce (*Picea abies* L.) experimental plots at 508 different locations in Southwest Germany.

Results The evaluation shows that periodic site index is statistically superior to the conventional site index, based on chronological stand age, for estimating stand height and stand volume increment. The analysis of temporal differences between growth-effective stand age and chronological stand age and between periodic site index and conventional site index in the period 1900 to 2020 reveals trends referring to stand age and site productivity, which corroborate earlier regional studies on forest growth trends due to environmental changes.

Conclusions The periodic site index is a better indicator for site productivity than conventional site index. Under conditions of environmental changes, conventional site index is biased, whereas the growth-effective age based site index provides an unbiased estimate of stand height development. With the more widespread application of remote sensing techniques, such as airborne laser scanning, the availability of multi-temporal stand height data will increase in the near future. The novel concept provides an adaptive modeling approach perfectly suited to these data for an improved estimation and prediction of forest site productivity under environmental changes and can straightforwardly be applied also to uneven-aged and multi-species stands.

Keywords Height-age model, Adaptive modeling, Growth trends, Climate change impact, Self-calibration, Norway spruce

Handling editor: John M. Lhotka.

*Correspondence: Chaofang Yue Chaofang.Yue@forst.bwl.de Full list of author information is available at the end of the article



© The Author(s) 2024. **Open Access** This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

1 Introduction

Estimation of forest site productivity is a critical issue for predicting forest stand growth and yield. Reliable estimates of forest site productivity are essential for the sustainable management of forest resources. Various phytocentric or geocentric approaches have been introduced to assess forest site productivity (Leary 1985; Skovsgaard and Vanclay 2008; Weiskittel et al. 2011; Bontemps and Bouriaud 2014). Among phytocentric approaches, the stand dominant (or top) height of a tree species at any base age, termed site index (SI), is commonly used in forest management planning as a measure of forest site productivity (Assmann 1970; Carmean 1972). Although site productivity potential may not be fully represented by site index, it is the most widely accepted method of estimating site productivity of evenaged mono-species forest stands (Skovsgaard and Vanclay 2008; Pretzsch 2009; Weiskittel et al. 2011).

For capturing between- and within-site class variation in data sets of stand height or stand height growth versus stand age (García 2011), Bailey and Clutter (1974) introduced the algebraic difference approach (ADA) with the property of base-age invariance for modeling the dominant height/site index. Cieszewski and Bailey (2000) further expanded parameter estimation techniques of base-age invariant SI models by developing the generalized algebraic difference approach (GADA), which exhibits additional properties such as polymorphism and variable asymptotes. More recently, the mixed-effects model is explored in site index modeling to estimate and predict local, site-specific height growth trajectories on different sites (Wang et al. 2007), silvicultural treatments (Fang and Bailey 2001), and climate and fertilizer applications (e.g., Wang et al. 2007). Advantageously, the mixed-effects model approaches can be used to fit models with both fixed and random effects parameters under addressing error structures, including a hierarchical, serially correlated error structure and heteroscedasticity.

In more complex uneven-aged multi-species stands, application of the dominant height site index for a selected tree species is more challenging, because a single meaningful age does not exist. Additionally, the probability that currently dominant trees may not have been dominant during their whole lifetime, but may have experienced suppressed growth in phases of canopy pressure in the past, is even higher than in even-aged mono-species stands. This can lead to an underestimation of site index (Monserud 1984; Magnussen and Penner 1996).

As an alternative, age-independent methods for quantifying site productivity have been described (Weiskittel et al. 2011), e.g., the growth intercept method, which estimates site index as a function of

internodal distances at some lower part of the stem (Wakeley and Marrero 1958). Although this method is directly applicable only to tree species that have recognizable internodes marking annual height growth, Nigh (1996) applied it to a species without this characteristic by using stem analysis data. However, applicability of such methods in forest practice is limited (Weiskittel et al. 2011; Riofrío et al. 2023).

Other age-independent measures such as the "site form" (Vanclay and Henry 1988) or the "site productivity index" (Huang and Titus 1993) have been studied in recent years (Fu et al. 2018; Moreno- Fernández et al. 2018; Molina-Valero et al. 2019). In fact, for a successful application of the site productivity index method, the following assumptions should be fulfilled: (i) decreasing tree taper is associated with increasing site productivity, and (ii) stand density does not affect the heightdiameter relationship of the dominant and codominant trees (Huang and Titus 1993). For the application of this method, an index diameter needs to be defined. This is a deficient property since diameter growth is sensitive to stand density (Wang 1998; Weiskittel et al. 2011).

Arias-Rodil et al. (2015) tested different methods of estimating site index and proposed an iterative method using height measurements derived from stem analysis data. However, their iterative method did not improve the estimation of height growth and site index, and the reliability of these methods has not been widely tested (Molina-Valero et al. 2019).

By contrast, geocentric approaches are based on the dependence of site productivity from soil, climatic, topographical, or ecological variables (Vanclay 1992). A large number of studies have thoroughly been conducted, although it has shown to be difficult to test them against the "true" site productivity (Daniel et al. 1979). Therefore, predictive accuracy of such site productivity models remains unsatisfactory (Bontemps and Bouriaud 2014). Another challenge for geocentric approaches is that they may not always be practical or affordable (Skovsgaard and Vanclay 2008).

For site index to be a measure of forest site productivity consistent over time, the existing methods and approaches for the estimation of forest site productivity described above take the assumption that environmental conditions remain stationary over time. For reasons of simplicity, it is still widely assumed that site productivity potential should be constant and invariant within site types that are uniform with respect to soil, climate, and topography. However, with the ongoing changes in environmental conditions, one has to recognize that site potential and, hence, forest site productivity are not constant over time. Several studies have shown that forest site productivity does not only vary spatially but temporally as well (Spiecker et al. 1996; Kahle et al. 2008; Skovsgaard and Vanclay 2013; Kohnle et al. 2014). Yue et al. (2014) introduced an approach to assess temporal changes in site index based on data from repeated measurements of Norway spruce experiments covering more than a century. Their results provided clear evidence that in Southwestern Germany site index has not been stable over time but has shown an increasing trend during the second half of the twentieth century.

In this context, the objective of this study is to present a novel concept for the assessment of site index and forest site productivity without prior information on forest age and applicable under conditions of environmental changes. This study is based on the following hypotheses: (i) using stand height data from repeated measurements a growth-effective stand age can be estimated based on a reference site index model; (ii) the growth-effective stand age can be used to determine the periodic site index; (iii) the periodic site index is a valid indicator for the site productivity of forest stands; (iv) changes over time in growth-effective stand age and periodic site index are indicative of changes in forest site productivity.

2 Material and methods

2.1 Data description

The database used in this study for the development of the new concept comprises periodically repeated individual-tree level measurements of 945 Norway spruce (*Picea abies* L.) experimental plots at 508 different locations (stands) in Baden-Württemberg, Southwest Germany (Table 1). Norway spruce was chosen, as the species is of major economic importance in Europe (Avitabile et al. 2020) and growth data are available from an extensive network of long-term experiments. All stands were even-aged and mono-species.

The plots cover a broad range with respect to elevation, precipitation, and temperature and, hence, in site indices (Table 1). The thinning treatments varied from unthinned dense stands, across low intensity thinnings from below, and high-intensity high thinnings in favor of selected future crop-trees, to open grown stands (i.e., almost solitary trees). Actually, the changes in thinning types and intensities applied on the plots mirror the historical development of forest thinning practices during the last century (e.g., Abetz and Klädtke 2002). However, although the thinning intensity of the experimental plots has increased by time, there are numerous studies giving evidence that tree and stand height growth, in contrast to diameter growth, is little affected by stand management (e.g., Mäkinen and Isomäki 2004; Mäkinen et al. 2005).

On each plot, the stem diameter at breast height (1.30 m) of all trees and the height of selected sample trees were measured repeatedly. On average, plots were measured 5.8 times in survey intervals of approximately 5 years. Based on the individual tree-level measurements, stand characteristics such as stand basal area, stand basal area increment, and mean stand diameter were calculated for each survey. Stand height curves were derived for each plot, which were then used to calculate the stand dominant height (H_{100} , syn. stand top height).

2.2 Growth-effective age

We present a "periodic site index" (pSI) based on stand dominant height and "growth-effective age" (eA) using repeated measurements of stand dominant height. For

Table 1 Descriptive statistics of Norway spruce experimental stands (n = 945) and trees

Parameter	Unit	Mean	Min	Max
Elevation	m asl	631.1	200	1246
Temperature (May–Sep)	°C	11.2	6.1	16.3
Precipitation (May–Sep)	mm	571	159	1613
Plot size	ha	0.21	0.02	1.1
Stand age	а	62	10	212
Number of trees	<i>n</i> /ha	1510	101	71,233
H ₁₀₀	m	24.6	3.6	42.6
Stand basal area	m²/ha	41.4	0.4	81.3
Stand volume	m³/ha	462.7	0.1	1404.6
TVP	m³/ha	626.3	0.1	2022.6
Survey year	а	1962	1872	2021
Period length	а	5.2	0.8	25
Number of surveys	n	5.8	2	19

H₁₀₀ Height of the mean basal area tree of the 100 thickest trees per hectare, TVP Total (accumulated) aboveground wood volume production (m³/ha)

concept development, we used data from periodically repeated measurements on long-term experimental plots but the method can likewise be applied to multi-temporal measurements of stand dominant height, e.g., in successive forest inventories or surveys.

Consider repeated assessments of stand dominant height and stand age, the mean height development of the dominant height trees follows a dominant height-age model, such as the one presented in Fig. 1.

The growth-effective age (eA) is an estimate of the growth developmental age of dominant height trees and is determined as follows: The stand dominant height at ages A_{t-1} and A_t is H_{t-1} and H_t , respectively. The height growth between A_{t-1} and A_t (i.e., the periodic height increment) can then be described by means of an algebraic difference approach (ADA) dominant height site index model. For the concept, we describe here, any initial height site index model can be used. We choose Sloboda's model (Sloboda 1971) due to its high statistical performance and its wide application in site index modeling in Germany (Nothdurft et al. 2012; Yue et al. 2016; Riedel et al. 2017). Sloboda's model is described by the following equation (Eq. 1):

$$H_{t} = 65^{\varnothing_{1}} \left(\frac{H_{t-1}}{65^{\varnothing_{1}}}\right)^{\exp\left(\frac{\varnothing_{2}}{(\varnothing_{3}-1)A_{t}}(\varnothing_{3}-1)} - \frac{\varnothing_{2}}{(\varnothing_{3}-1)A_{t-1}}\right)}$$
(1)

where $Ø_1$, $Ø_2$, and $Ø_3$ are model parameters.

If A_{t-1} and A_t are replaced by eA and eA + p in equation (Eq. 1), the following equation is obtained:

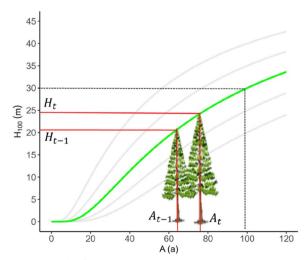


Fig. 1 Example of a stand dominant height-age model (H_{100} : stand dominant height, *A*: stand age, *t*: number of the survey). The green curve corresponds to a site index of 30 (m) at base age 100 years

$$H_{t} = 65^{\varnothing_{1}} \left(\frac{H_{t-1}}{65^{\varnothing_{1}}}\right)^{\exp\left(\frac{\varnothing_{2}}{(\varnothing_{3}-1)(eA+p)}(\varnothing_{3}-1)} - \frac{\varnothing_{2}}{(\varnothing_{3}-1)eA}(\varnothing_{3}-1)}\right)$$
(2)

where *p* is the interval length in years between two successive measurements (i.e., between survey year SY_{t-1} and SY_t). Since *p* is known, and the model parameters of the initial height site index model (Eq. 1) are given, the only unknown variable in Eq. 2 is *eA*, which can then be estimated from the input variables stand dominant heights H_{t-1} and H_t , and the interval length in years *p* by using a statistical optimization procedure as follows:

$$eA := \underset{eA \in [10,200]}{\operatorname{argmin}} f(eA|H_{t-1}, H_t, p; \varnothing)$$
$$= \underset{eA \in (10,200]}{\operatorname{argmin}} \left(H_t - 65^{\varnothing_1} \left(\frac{H_{t-1}}{65^{\varnothing_1}} \right)^{\exp\left(\frac{\varnothing_2}{(\varnothing_3 - 1)(eA + p)} (\varnothing_3 - 1)} - \frac{\varnothing_2}{(\varnothing_3 - 1)eA (\varnothing_3 - 1)} \right) \right)$$
(3a)

Hence, eA is a function of initial stand dominant height in SY_{t-1} and stand dominant height in SY_t for an interval (period) of length p (we set eA to range from 10 to 200 years). Owing to the one-dimensional optimization problem, we use function "optim" with method "brent" of the package stats in R 4.1.3 (R Core Team 2023).

Equation 3a can be rewritten as follows:

$$eA := f(H_{ini}, \Delta H, p; \varnothing)$$
(3b)

 H_{ini} is the initial stand dominant height $(H_{ini} = H_{t-1})$ and ΔH is stand dominant height increment $(\Delta H = H_t - H_{t-1})$.

It can be seen that the initial stand dominant height at SY_{t-1} is clarified from that time onwards, e.g., the site trees are free from suppression, damages, and disease. As the initial dominant height and the length of the survey period are known, change in growth of stand dominant height (ΔH) in the surveyed period is decisive for the growth-effective age. To distinguish growth-effective stand age from the observed stand age, observed stand age is hereinafter referred to as chronological stand age.

2.3 Periodic site index

Based on growth-effective age (*eA*), the periodic site index (*pSI*) within survey periods ($p = \sum_{l=1}^{l} p_l$) is formulated as

$$pSI = f(H, eA, \boldsymbol{p}; \varnothing)$$
(4)

with \varnothing a vector of the same model parameters as in Eq. 1.

The estimate of eA based only on data from a single period (p), however, does not warrant a stable estimate of eA, since the height increment in a given period may also be subject to annual and/or periodic fluctuations.

Therefore, we introduce the periodic site index (*pSI*), which is estimated according to the following multi-step procedure (Fig. 2):

- 1) If repeated or multi-temporal measurements of stand dominant height are available, the corresponding survey years (SY_t) and the respective interval lengths (p_t) are known as well.
- 2) Starting from the first pair of dominant height measurements at the first and second survey year $(SY_1 \text{ and } SY_2)$ together with the interval length p_1 , the growth-effective age for the first period (eA_1) is estimated by using the optimization procedure given in Eq. 3a.
- 3) Step 2 is repeated until estimates of growth-effective age $(e\dot{A}_t)$ are available for all survey periods. To obtain a robust and stable estimate for the mean growth-effective age, a reference year is specified. For convenience, we defined the calendar year of SY_1 as reference year. Then, the corresponding growth-effective ages $(e\dot{A}_1^t)$ are inferred for all measurement intervals with respect to this reference year according to Eq. 5.

$$e\dot{A}_{1}^{t} = e\dot{A}_{t} - \sum_{i=1}^{t-1} p_{i}$$
(5)

4) The mean growth-effective age in the first survey year (SY_1) of a stand (or plot) is then calculated as the robust mean of all growth-effective age estimates using Tukey's biweight robust mean (Tb) (Mosteller and Tukey 1977):

$$\overline{e\dot{A}_1} = Tb(e\dot{A}_1^t) = \frac{\sum_{i=1}^t \psi(\mu_i) \cdot e\dot{A}_1^i}{\sum_{i=1}^t \psi(\mu_i)}$$
(6)

Tukey's biweight is defined as follows:

$$\psi(\mu_i) = \begin{cases} \mu_i (1 - \mu_i^2)^2 & |\mu_i| \le 1\\ 0 & otherwise \end{cases}$$
(7)

$$\mu_i = \frac{e\dot{A}_1^i - \overline{e\dot{A}_1}}{cs} \tag{8}$$

where *s* is the median and *c* is a constant. Tukey's biweight robust mean is calculated using function "tbrm" in the package dplR (Bunn et al. 2023) in R 4.1.3 (R Core Team 2023).

5) Subsequently, the growth-effective age for each survey year can be derived from the weighted mean growth-effective age in the reference year (Eq. 6) and the lengths of the corresponding measurement intervals as follows:

$$eA_t = \overline{e\dot{A}_1} + \sum_{i=1}^{t-1} p_i \tag{9}$$

6) The individual period-specific periodic site index (*pSI_t*) is then calculated based on the growth-effective age and dominant height in survey year *SY_t*. Finally, the mean periodic site index (*pSI*) of each plot is averaged using Tukey's biweight mean in the periods spanning from *SY*₁ to *SY_t*.

Calculation step	Iteration process	Formulae
Data: SY, H	$\xrightarrow{SY_1 SY_2 SY_3 SY_{t-1} SY_t}$	
	p_1 p_2 \cdots p_{t-1}	$p_{t-1} = SY_t - SY_{t-1}$
	(H_1)	$e\dot{A}_{t-1} = \underset{eA \in \{10, 200\}}{\operatorname{argmin}} (f(e\dot{A}_t p_t, H_{t-1}, H_t; \emptyset))$
eA	$e\dot{A_1} e\dot{A_1} e\dot{A_1} e\dot{A_2} \cdots e\dot{A_{t-1}} e\dot{A_{t-1}}$	$e\dot{A}_{1}^{t} = e\dot{A}_{i} - \sum_{l=1}^{t-1} p_{l}$
		$\overline{e\dot{A}_1} = Tb(e\dot{A}_1^t)$
	eA_1 eA_2 eA_3 \cdots eA_{t-1} eA_t	$eA_t = \overline{e\dot{A}_1} + \sum_{l=1}^{t-1} p_l$
	pSI_1 pSI_2 pSI_3 \cdots pSI_{t-1} pSI_t	$pSI_t = f(H_t, eA_t; \emptyset)$
pSI	pSI	$pSI = Tb(pSI_t)$
Por l	pSI	$pSI = Tb(pSI_t)$

Fig. 2 Process diagram for the estimation of growth-effective age (*eA*) and periodic site index (*pSI*) based on repeated or multi-temporal stand dominant height measurements in survey periods spanning from SY_1 to SY_t

2.4 The properties of growth-effective age and periodic site index

Periodic site index (*pSI*) is estimated from growtheffective stand age (*eA*) and stand dominant height, analogous to the conventional site index (*SI*) which is estimated from chronological stand age (*A*) and stand dominant height. Under stationary environmental conditions, *eA* is equivalent to *A*, and, following stochasticity, the differences between *eA* and $A(\Delta A = eA - A)$ will fluctuate randomly around zero. In a series of multiple surveys, the sequence of $\Delta A = \{\Delta A_t : t \in T\}$ can be considered a stochastic stationary process with $\Delta A_t \sim N(0, \sigma_{\Delta A}^2)$.

Correspondingly, under stationary environmental conditions, *pSI* is equivalent to *SI*, and the differences between *pSI* and $SI(\Delta SI = pSI - SI)$ will also fluctuate around zero and can be considered a stochastic stationary process with $\Delta SI_t \sim N(0, \sigma_{\Delta SI}^2)$.

However, under temporally changing environmental conditions, site productivity may exhibit non-stationarity as well. For example, in the case of persisting growth stimulating environmental effects on forest growth, trees might grow faster than expected, i.e., aging decelerates and trees perform like chronologically younger trees. In such a situation, eA < A, and the difference between eA and A is less than zero ($\Delta A < 0$). Correspondingly, site quality is improved, pSI > SI, and the difference between pSI and SI is greater than zero ($\Delta SI > 0$).

However, if persisting negative environmental effects on forest growth occur, trees grow slower than expected, i.e., aging accelerates and trees perform like chronologically older trees. In such a situation, eA > A, and the difference between eA and A is greater than zero ($\Delta A > 0$). Correspondingly, site quality of the stand is deteriorated, pSI < SI, and the difference between pSI and SI is smaller than zero ($\Delta SI < 0$).

Hence, over multiple surveys, the sequence of $\Delta A = \{\Delta A_t : t \in T\}$ can be considered a stochastic nonstationary process with $\Delta A_t \sim N(f_{\Delta A}(t), \sigma^2_{\Delta A})$, and correspondingly, the sequence of $\Delta SI = \{\Delta SI_t : t \in T\}$ a stochastic non-stationary process with $\Delta SI_t \sim N(f_{SI}(t), \sigma^2_{\Delta SI})$.

By theory, growth-effective age (eA) should be a better measure for the growth vitality and developmental stage of the trees than chronological age, and growth-effective age based periodic site index (pSI) a better measure for site productivity than conventional site index.

2.5 Statistical evaluation

For the evaluation of *eA* and *pSI*, we first conduct a direct comparison of *eA* and *A*, with *SI* and *pSI*, respectively. Secondly, we design height and volume increment models,

Page 6 of 18

which include the relevant predictor variables, i.e., initial size, competition, and site index, and then evaluate the contributions of *SI* and *pSI* for predicting stand height and stand volume growth. We use the following model for stand height increment (ΔH)

$$\Delta H = f(\mathbf{H}, SDI, SI; \boldsymbol{\alpha}) = \alpha_1 \cdot H^{\alpha_2} \cdot e^{(\alpha_3 \cdot H + \alpha_4 \cdot SDI + \alpha_5 \cdot SI)}$$
(10)

and for stand volume increment (ΔV)

$$\Delta V = f(V, SDI, SI; \alpha) = \alpha_1 \cdot V^{\alpha_2} \cdot e^{(\alpha_3 \cdot V + \alpha_4 \cdot SDI + \alpha_5 \cdot SI)}$$
(11)

where *H* is initial stand dominant height (m), *V* is initial stand volume (m³/ha), *SDI* is Reineke's Stand Density Index (Reineke 1933), SI = (SI, pSI), and $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_5)$ are model parameters.

Furthermore, we consider the mean annual increment in total stand volume production (MAI), derived from the total (accumulated) aboveground wood volume production (TVP), as the best measure of site productivity. MAI_{max} is often used and refers to the MAI at the stand age of its culmination. However, the stand age at which this happens may vary greatly, even within a tree species, depending on site quality and stand treatment (Assmann 1970). Because of that, MAI at a pre-defined reference age has some advantage and is in practical forest management planning preferred over MAI_{max} (Assmann 1970). For example, MAI_{100} refers to the mean annual increment in total stand volume at the reference age of 100 years.

Given repeated or multi-temporal measurement data of stand dominant height and chronological stand age, a conventional dominant height site index model can directly be estimated. Hence, it needs to be tested whether the periodic site index model based on the dominant height and the growth-effective age (eA) can be estimated accordingly and how both models compare. For that purpose, Sloboda's ADA dominant stand height site index model (Sloboda 1971) is estimated for both chronological stand age and growth-effective stand age. The data structure for all nonoverlapping, ascending growth intervals with $SY_t < SY_{t+1}$, provides all necessary growth information for unbiased and efficient estimates of the model parameters (Wang et al. 2004; Yue et al. 2016). Continuous-time auto-regression CAR(1) (Pinheiro and Bates 2000) is used to account for the inherent autocorrelation of the longitudinal data, which enables the model to be applied also to unevenly spaced and unbalanced data (Gregoire et al. 1995). To avoid the possible problem of heteroscedasticity, the variance of errors was assumed to be a power function of the predicted dominant height $(Var(\varepsilon) = \sigma^2 \hat{H}^{2\delta})$. The correction for autocorrelation and heteroscedasticity is necessary to obtain unbiased and efficient estimates of the model parameters. Parameters of the height/site index model are finally estimated using generalized non-linear least squares (GNLS) implemented in the package nlme (Pinheiro and Bates 2000) in R 4.1.3 (R Core Team 2023).

2.6 Updating the site index model

For the effective application of the described novel concept, a valid site index (SI) model, i.e., height-age model, is indispensable. However, in practice, any given site index model might be outdated or a site index model for a specific tree species is not available. Under these conditions, the used *SI* is subject to bias and might not accurately reflect the current dynamics of stand height growth versus stand age. We therefore suggest a recursive procedure for updating, or self-calibrating, the height site index model based on repeated or multi-temporal measurements of stand dominant height (Fig. 3).

The estimation process consists of the following steps:

- 1) Estimate the growth-effective age (eA^i) based on SI^i and H according to the multi-step procedure described above based on an existing site index model $(SI^i)_{i=0}$ and repeated measurements of stand dominant height (H).
- 2) Estimate the new site index model (SI^{i+1}) based on the estimated growth-effective age and repeated measurements of stand dominant height (H).
- 3) Calculate the following stand height increment model (Eq. 9) by using SI^i and SI^{i+1}

$$\Delta H_i = f(H, SDI, SI^i; \boldsymbol{\alpha})$$

then ΔAIC_i is obtained based on the two successive AIC_i and AIC_{i+1}

$$\Delta AIC_i = AIC_{i+1} - AIC_i$$

with $AIC_i = AIC(\Delta H_i)$.

4) Check if ΔAIC_i is larger than a positive value ε , if $\Delta AIC_i > \varepsilon$, steps 1 to 3 are iteratively repeated to go through a new loop $(i \leftarrow i + 1)$ of estimating growth-effective stand age (eA^i) and then construct the new site index model (SI^{i+1}) until ΔAIC_i is less than ε , and finally select the updated site index model (SI^{up}) with the smallest value of AIC.

3 Results

3.1 Growth-effective stand age (eA) and periodic site index (pSI)

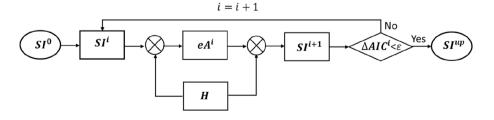
For the first evaluation step, growth-effective stand age (eA) and periodic site index (pSI) are compared with the chronological stand age (A) and the conventional site index (SI), respectively (Fig. 4).

The left graph in Fig. 4 shows the scatter plot of *A* and *eA*; the right graph shows the scatter plot of *SI* and *pSI*. In both cases, the regression lines point to highly significant relations. The coefficient of determination (R^2) of *SI* and *pSI* is highly significant, but lower than for the age estimate. The comparisons indicate considerable statistical variation for *A* and *eA* (Fig. 4 left) and even more for *SI* and *pSI* (Fig. 4 right).

3.2 Differences between eA and A, and pSI and SI versus calendar year

For the second evaluation step, we analyze time series of differences between eA and A, and between pSI and SI versus calendar year, using the period 1900 (before 1900 data is sparse) to 1940 as reference period (Fig. 5).

During the period 1900 to 1940, the level of ΔA is in quasi-steady state. Afterwards, it shows a decreasing trend with a minimum of -7.5 years (i.e., trees behave like being 7.5 years younger in comparison to the



 $(i = 0, 1, 2, \cdots, n)$

Fig. 3 Process diagram for the recursive updating of the site index model. SI^0 is the initial site index model, SI^i is the estimated *i*th site index model, and SI^{up} is the recursively updated final site index model

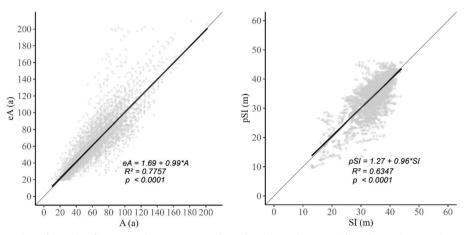


Fig. 4 Bivariate scatterplots of growth-effective stand age (*eA*) versus chronological stand age (*A*) (left) and periodic site index (*pSI*) versus conventional site index (*SI*) (right). Thin lines: bisecting line, thick lines: linear regression line

reference period 1900–1940) around 1975, followed by a slight increase until around 2000, and from that on it remains approximately on the same level until 2020.

Correspondingly, from 1900 to 1940, the level of ΔSI is in a quasi-steady state, but then changes continuously until a maximum of +2.5 m (i.e., trees show an improvement in site index of 2.5 m) at around 1975. Since then, ΔSI shows a slight decrease until the beginning of 2000 and from then on remains approximately on the same level until 2020.

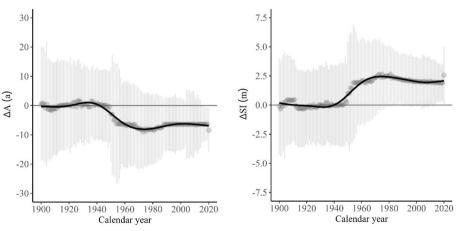
3.3 Dominant height increment and stand volume increment

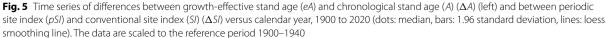
The statistical relations between the periodic mean annual increment in dominant height and in stand volume with *SI* and *pSI* respectively are used to evaluate

the performance of the novel approach against the conventional approach.

The left graph in Fig. 6 shows the relation between the periodic mean annual increment in dominant height (ΔH_{100}) and *SI* based on the chronological stand age (R^2 of the power regression line is 0.29, *p* < 0.0001). The right graph in Fig. 6 shows the relation between the periodic mean annual increment in dominant height (ΔH_{100}) and *pSI* based on the growth-effective stand age (R^2 of the power regression line is 0.35, *p* < 0.0001). When using *pSI* instead of *SI*, the coefficient of determination of the relationship increases by more than 20%.

The left graph in Fig. 7 shows the relation between the periodic mean annual stand volume increment (ΔV) and the height site index (*SI*) derived on the basis of





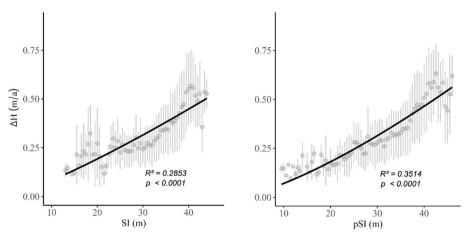


Fig. 6 Relation between the periodic mean annual increment in dominant height (ΔH) and conventional site index (*SI*) (left), and periodic site index (*pSI*) (right), respectively (dots: median, bars: 1.96 standard deviation, lines: power regression line)

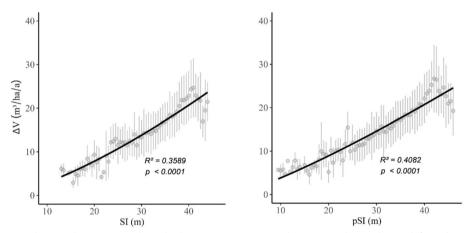


Fig. 7 Relation between the periodic mean annual stand volume increment (ΔV) and conventional site index (*SI*) (left), and periodic site index (*pSI*) (right), respectively (dots: median, bars: 1.96 standard deviation, lines: power regression line)

chronological age (R^2 of the power regression line is 0.36, p < 0.0001). The right graph in Fig. 7 shows the relation between the periodic mean annual stand volume increment and the periodic site index (*pSI*) derived on the basis of the growth-effective age (R^2 of the power regression line is 0.41, p < 0.0001). For *pSI*, the coefficient of determination is larger by more than 14%.

3.4 Model-based prediction of stand height increment and stand volume increment with site index (*SI*) and periodic site index (*pSI*)

To compare the contributions of *SI* and *pSI* to the prediction of the periodic mean annual stand dominant height increment and stand volume increment, we formulated two models with identical structure. The stand dominant height increment is modeled as a function of initial stand dominant height, Reineke's Stand Density Index (*SDI*, Reineke 1933), and the site index (*SI* or *pSI*).

Table 2 summarizes the statistical characteristics of the two models for the periodic mean annual stand dominant height increment model (Eq. 10). The comparison shows that the *pSI*-based model is superior to the *SI*-based model in all statistical parameters (tvalue, R^2 , *RMSE*, and *AIC*). Analogous to the models for stand dominant height increment, stand volume increment is modeled as a function of initial stand volume, *SDI*, and *SI* or *pSI*, respectively. Table 3 summarizes the statistical characteristics of the two stand volume increment models (Eq. 11). The comparison of the two models again shows that the *pSI*-based model is superior to the *SI*-based model in terms of all statistical parameters (t-value, R^2 , *RMSE*, and *AIC*).

SI	Variables	Variables		t-value	Pr(> t)	R^2	RMSE	AIC	n
	Parameter	Estimate							
SI	α_1	0.02476	0.00229	10.813	< 0.0001	0.746	0.097	-9883	5435
	α_2	0.99061	0.04768	20.775	< 0.0001				
	α_3	-0.09421	0.00257	-36.728	< 0.0001				
	α_4	-0.00005	0.00001	-8.982	< 0.0001				
	α_5	0.05339	0.00093	57.660	< 0.0001				
pSI	α_1	0.03722	0.00286	13.012	< 0.0001	0.816	0.083	-11,639	5435
	α_2	0.82060	0.04111	19.961	< 0.0001				
	α_3	-0.08638	0.00220	-39.289	< 0.0001				
	α_4	-0.00003	0.00001	-6.949	< 0.0001				
	α_5	0.05288	0.00067	79.080	< 0.0001				

Table 2 Summary statistics of the periodic mean annual stand dominant height increment model (Eq. 10) as a function of conventional site index (*SI*) and periodic site index (*pSI*) (R^2 , coefficient of determination

RMSE Root mean squared error, AIC Akaike information criterion, n Degrees of freedom

Table 3 Summary statistics of the periodic mean annual stand volume increment model (Eq. 11) as a function of conventional site index (*SI*) and periodic site index (*pSI*)

SI	Variable	Variable		t-value	Pr(> t)	R^2	RMSE	AIC	n
	Parameter	Estimate							
SI	α_1	1.04600	0.07968	13.128	< 0.0001	0.46	4.765	32,347	5435
	α_2	0.13780	0.01362	10.117	< 0.0001				
	α_3	-0.00062	0.00004	-14.286	< 0.0001				
	α_4	0.00017	0.00001	25.550	< 0.0001				
	α_5	0.05277	0.00097	54.514	< 0.0001				
pSI	α_1	1.67917	0.11231	14.952	< 0.0001	0.51	4.517	31,768	5435
	α_2	0.10858	0.01283	8.461	< 0.0001				
	α_3	-0.05277	0.00376	-14.035	< 0.0001				
	α_4	0.00017	0.00001	27.508	< 0.0001				
	α_5	0.04735	0.00076	61.907	< 0.0001				

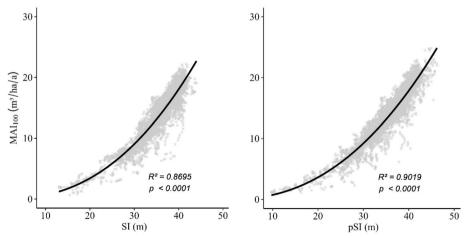


Fig. 8 Relation between mean annual increment in stand total volume at reference age 100 years (MAI_{100}) and conventional site index (SI) (left) and periodic site index (pSI) (right) (dots: measurement data, lines: power regression line)

3.5 Mean annual increment in total stand volume (MAI)

The two graphs in Fig. 8 illustrate the relations between measured MAI_{100} and SI and pSI, respectively. The left graph shows the relationship between measured MAI_{100} and SI (R^2 of the power regression line is 0.87, p < 0.0001). The right graph shows the relationship between measured MAI_{100} and pSI. R^2 is 0.90 and thus slightly higher than its counterpart based on SI.

3.6 Comparison of SI and pSI

For the comparison of SI and pSI, we focus on two key questions: 1. Can a suitable site index model be constructed based only on data of repeated measurements of stand dominant height without considering the chronological stand age? 2. What is the statistical performance of such a model compared to the conventional site index model? To answer these questions, we estimated the conventional site index (*SI*) based on the chronological stand ages (Eq. 1) and the periodic site index (*pSI*) based on growth-effective age (Eq. 4). The parameter estimates of *SI* and *pSI* and the fit statistics are presented in Table 4. All parameters are significant. Both models explained more than 99% of the variation in the response variable. For validation, residuals of the chronological age based site index model and the growth-effective age based site index model are plotted versus the independent variables, the predicted variables, and the calendar years. Both models produce residuals almost randomly distributed around zero with homogeneous variance. No significant trends across stand age and predicted dominant height can be detected, except at juvenile and at older ages with fewer numbers of observations (Figures 11, 12, and 13 in Appendix).

However, residuals plotted versus calendar year clearly show that the residuals were negative from 1890 until around 1955 and from around 1965 mean residuals became positive until recent years (Fig. 9), i.e., the *SI* model overestimated stand dominant height growth before around 1960 whereas underestimated it after that. In contrast, residuals of the *pSI* model do not show detectable trends from 1880 to 2020, i.e., the *pSI* model provided unbiased estimates of stand height growth.

3.7 Updating the site index model

To evaluate the recursive procedure for updating (or self-calibrating) the height site index model, we parameterized the height site index curves of the Norway

Table 4 Parameter estimates and goodness-of-fit statistics of SI (Eq. 1) and pSI (Eq. 4)

SI	Para	Estim	Std.Error	t-value	Pr(> <i>t</i>)	R ²	RSME	AIC	ρ	δ
SI	Ø1	0.9746	0.0041	239.69	< 0.0001	0.996	0.5045	7999	0.81	-0.57
	Ø2	0.7030	0.0264	26.59	< 0.0001					
	Ø3	0.9201	0.0134	68.48	< 0.0001					
pSI	Ø1	0.9586	0.0024	392.12	< 0.0001	0.997	0.4273	6193	0.59	-0.70
	Ø2	0.6630	0.0190	34.84	< 0.0001					
	Ø ₃	0.8812	0.0096	92.05	< 0.0001					

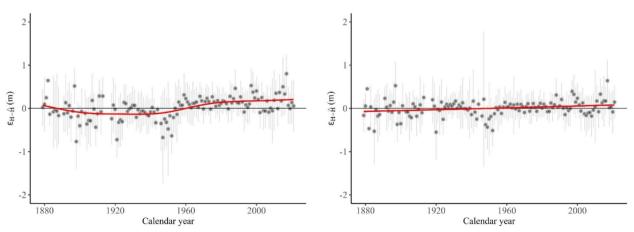


Fig. 9 Residuals of the conventional site index model (Eq. 1) (left) and the growth-effective age based site index model (Eq. 1) (right) versus calendar year (dots: median, bars: 1.96 standard deviation, lines: loess smoothing line)

spruce yield table by Assmann and Franz (1965) with Sloboda's function (Eq. 1), which we refer to as outdated site index model. We updated this outdated model based on the iterative, self-calibration procedure by using repeated measurements of stand dominant heights (see Section 2.6). Figure 10 shows the height-age curves (site index curves) estimated directly based on the repeated measurements of stand dominant height (Eq. 1), derived from the outdated Norway spruce yield table by Assmann and Franz (1965), and updated *SI* based on Assmann and Franz, respectively. In this case, the updated *SI* is nearly identical to the *SI* estimated from measured height-age data.

4 Discussion

The height site index concept is based on the height-age relationship. In principle, the estimation of height site index is based on the following assumption: dominant height growth only depends on stand age and site quality, not on stand density. From theory an unbiased estimation of stand height growth/site index from chronological stand age is possible if two other assumptions are fulfilled:

- The selected site trees belonged to the crown class predominant or dominant throughout their lifetime and were not exposed to any competition, damage, or disease harmful to their height growth.
- Stand height growth has been in equilibrium with environmental conditions, i.e., environmental (cli-

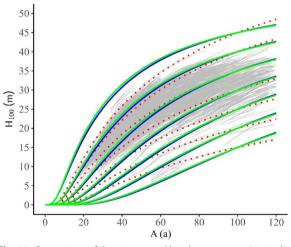


Fig. 10 Comparison of dominant stand height-age curves (site index curves) of the outdated *SI* (red dots) (Eq. 1; Assmann and Franz 1965) and actual *SI* (blue lines) estimated based on repeatedly measured height-age data (Eq. 1), as well as updated *pSI* (green lines) using the recursive procedure (Eq. 4), and measured dominant heights (gray lines)

matic) fluctuations and long-term changes at a particular site were negligible and had no decisive impact on height growth.

The first assumption is a necessary characteristic for site trees with regard to describing the past development of height growth. Since, in reality, trees in a stand are subject to stand dynamics and all kind of changes in environmental conditions, these assumptions are fraught with considerable uncertainties. This is especially true if no information on the height growth development of the site trees is available for the time before the first survey, e.g., the dynamics of stand dominant height related to stand-level tree mortality and changes in social status (Raulier et al. 2003).

Equally critical is the third assumption, namely that the site trees grow under stationary environmental conditions. The results obtained in this study (Figs. 5 right and 9 left) clearly indicate that time series of site indices are not necessarily stationary, but can show significant trend-like changes. This is supported by earlier studies which found trends in site productivity in European Forests since at least the middle of the twentieth century (Spiecker et al. 1996; Kahle et al. 2008; Kohnle et al. 2014; Yue et al. 2014). Although in this study, based on the given database, evaluation of the contribution of SI and pSI in the stand dominant height and stand volume increment model does not show large differences (Tables 3 and 4), and bias was small and not significant in the model estimation (Fig. 9 left), the projected dominant height at reference age has a decisive impact on the ascertainment of site index (differences in site index of ca. 2.5 m, see Fig. 5 right).

Goelz and Burk (1998) applied an ADA dynamic height growth model to height-age data for their analysis of trends in height growth. Similarly, Martin-Benito et al. (2008) selected a GADA polymorphic model with variable asymptotes (Cieszewski and Bailey 2000). In these studies, ADA and GADA have been applied to construct dominant height/site index models in the context of environmental changes. Furthermore, autocorrelation and heteroscedasticity of the error-terms were considered during model fitting. The resulting models provided highly significant results, and conventional analysis of model residuals indicated no evidence of bias, autocorrelation, or heteroscedasticity when analyzed in respect to predicted stand dominant height and age (Goelz and Burk 1998; Martin-Benito et al. 2008). However, when model residuals were analyzed versus calendar year, Goelz and Burk (1998) identified a period of superior growth in model residuals

and Martin-Benito et al. (2008) detected synchronous trends in model residuals for the study regions.

The presence of time trends in model residuals is indicative of a failure of the applied *SI* models which are based on height-chronological age data to correctly capture calendar year related temporal variation in the data, no matter if ADA or GADA is applied. Hence, under nonstationary environmental conditions, the estimation and prediction of site productivity based on height-chronological age site index is biased. Even when data spanning wide ecological gradients and extending over more than a century of measurements are used, this is not a guarantee that all within- and between-site class variability is correctly reflected and captured even by advanced conventional site index models.

In contrast to the height-chronological age based models, the residuals of the growth-effective age based periodic site index (pSI) do not show significant deviations from expected values (Fig. 9 right). Because of the adaptive, periodic updating of pSI, deviations from the expected height-age trajectories are transient and much smaller.

In conclusion, if the last assumption is fulfilled, *pSI* provides a similar estimation as *SI* based on chronological stand age ($pSI \approx SI$). However, if the above two assumptions are violated, the *SI* estimate is biased, whereas *pSI* is capable to estimate forst site productivity with almost no bias.

In fact, eA has been used more frequently to estimate the potential dominant height increment (Hann and Ritchie 1988) or tree crown recession (Hann and Hanus 2004), and there it is defined as the age of a dominant tree with the same height and site index (*SI*):

$$eA := f(H, SI; \emptyset)$$

eA can be estimated based on the measured dominant height if the site index of a given stand is known. However, our aim was to estimate the site index of a given stand using eA and not to assume it as already known. To address these challenges, we defined a new growtheffective age (eA) (Eqs. 3a and 3b), which is determined by initial stand dominant height (H_{ini}) , increment in stand dominant height (ΔH), and the length of the surveyed interval $(p = SY_t - SY_{t-1})$. The site trees are required to be stand dominant (top) height trees. Changes in growth of stand dominant height in the surveyed periods result from the interaction of initial stand dominant height and environmental conditions during the surveyed periods. Under these conditions, assessment methods based on periodic increment perform better than methods based on cumulative performance (e.g., estimates based on the height-age relationship) (Skovsgaard and Vanclay 2013). As a consequence, this renders *eA* sensitive to growth dynamics of trees and stands and the *eA* based *pSI* is capable to capture dynamic stand site productivity.

In contrast, SI represents productivity retrospectively as an average value over the lifetime of the stand until to date. However, this carries the risk that the estimation of conventional site index (SI) for stands of different ages on the same site can lead to considerably different results. This is especially true when long-term environmental changes have occurred (Yue et al. 2014). As causes for changes in the productivity of Central European forests, several likely causes have been discussed: (i) improved tree nitrogen nutrition, due to anthropogenically increased nitrogen deposition and due to the recovery of forest sites from former misuse, (ii) longer growing seasons due to increased air temperatures, and (iii) CO₂ fertilization (e.g., Kahle et al. 2008; Etzold et al. 2020). More recent studies provide evidences that sign and magnitude of trends in the productivity of European forests and also their underlying causes show considerable variation in space and time (e.g., Etzold et al. 2020; Henttonen et al. 2024; Pretzsch et al. 2014, 2023; Ols et al. 2020).

Under such conditions, *pSI* can be applied, e.g., for site productivity mapping based on data from national forest inventories (NFI) for specific periods. Furthermore, by the analysis of temporal changes of ΔA and ΔSI , it is possible to directly detect and quantify dynamics of site productivity (Fig. 4).

In the statistical evaluation, the chronological stand age (A), and accordingly, SI are usually applied as reference measures. In our study, the direct comparison of A and eA, and of SI and pSI showed relatively large bias and variation (Fig. 4). From that, it could be inferred that *pSI* might not be a good indicator for stand growth and site productivity. In fact, the chronological stand age (A) cannot reflect the real growth development stage under changing environmental conditions (Fig. 4) and the derived SI is biased (Fig. 4). Therefore, in a further validation step, we designed a validation approach for comparing the contributions of SI and pSI to the prediction of stand height and stand volume growth. This validation demonstrated that *pSI* (based on growth-effective age) is statistically superior to the conventional site index (SI; based on chronological age) for the estimation of stand dominant height (Table 2), stand volume growth (Table 3), and mean annual total volume production (MAI_{100}) (Fig. 8). The additional statistical evaluation confirms the superiority of *pSI* over *SI* (see Table 5 in the Appendix), especially under changing environmental conditions. Therefore, pSI (periodic site index) is thus proven a management-relevant and more precise period-specific productivity estimator superior to the conventional site index (*SI*) and can be considered a generalized site index that is able to capture spatial and temporal variation in forest site productivity.

Our novel concept for the estimation of pSI based on eA has two mandatory requirements: periodically repeated or multi-temporal height measurements and an existing site index model suitable for the specific tree species. Repeated or multi-temporal measurements of stand dominant height are often available in practice today, for example, from terrestrial inventories such as stand-level, regional-level, or national-level inventories, which provide measurement data on tree/ stand heights (Kangas and Maltamo 2006; Tomppo et al. 2010). In case where there is no suitable site index model available, the recursive procedure presented in this study (Section 2.6) can be used for updating an initial site index model (Fig. 3). In fact, the basic idea behind the concept is self-calibration, i.e., estimation of *pSI* by using an existing site index model and new measurements of repeated stand dominant heights.

In addition, it is quite likely that in the future due to wider application of remote sensing technologies an increasing amount of multi-temporal measurement data on periodic changes in tree/stand height will become available (Tompalski et al. 2021), which are ideally suited for determining periodic site index based on growtheffective age. In fact, techniques such as airborne laser scanning (ALS) can already be routinely applied to assess tree height, often with higher accuracy than conventional terrestrial surveys (Socha et al. 2017; Tompalski et al. 2019; Noordermeer et al. 2020; Hawryło et al. 2024).

Evaluation of site quality in uneven-aged and multispecies stands is a great challenge for forest growth and yield modeling. The application of *SI* in uneven-aged forests is not reasonable because of the initial suppression of regenerating trees, especially shade tolerant species, and because of the unknown or meaningless tree/stand age (Burkhart and Tomé 2012). For *pSI*, only repeated measurements of stand dominant height are required, e.g., the selected site trees need to represent stand dominant height from the time of the first survey onwards. *pSI* only depends on the initial stand dominant height and the height growth in the subsequent survey periods.

Site form (SF) is recently explored to evaluate site quality in even- and uneven-aged stands (Fu et al. 2018; Molina-Valero et al. 2019). Application of the SF concept is built on the disputed assumptions of the dependence of tree taper on site productivity, and the independence of site form from stand density. Additionally, application of the SF concept requires measurements of stand dominant height and diameter, whereas stand diameter cannot accurately be measured using remote sensing technologies. For *pSI*, only repeated measurements of stand dominant height are needed, which can easily and accurately be measured using remote sensing technologies. Therefore, *pSI* is suitable for large-scale monitoring purposes.

The *pSI* is built on less assumptions than *SI* and SF and provides new ways to evaluate site quality in unevenaged stands. Due to stand dynamics, the collective of the stand dominant height trees is more prone to changes in uneven-aged stands than in even-aged stands. Single top height trees or an identical collective of top height trees can then be selected as site trees, instead of using stand dominant height (H_{100}) as in even-aged stands. Consequently, *pSI* can straightforwardly be applied in unvenaged (multi-aged) stands if repeated measurements of top height trees are available.

In summary, the concept of periodic site index based on stand dominant height–growth-effective age is appealing and could find a wide application for growth and yield modeling, especially under environmental changes.

5 Conclusions

We introduced the periodic site index *pSI* for quantification of site productivity, which possesses the following desirable main features:

- Tree/stand age-independent
- Detection and quantification of dynamic forest site productivity under changing environmental conditions
- Providing a better estimation and prediction of the height and volume growth of stands

The successful application of *pSI* requires (i) accurate measurements of stand dominant height and (ii) an existing height site index model suitable for the tree species in question. These two requirements can more easily be fulfilled in practice when remote sensing techniques, such as airborne laser scanning (ALS), are applied together with the iterative procedure presented to update existing height site index curves using multi-temporal dominant height measurements.

Therefore, the novel concept of pSI can directly be applied to multi-species, and/or uneven-aged stands, and to local-, regional-, and national-scale assessments of forest site productivity in multi-temporal ALS-based forest inventories (Noordermeer et al. 2020; Tompalski et al. 2021). Furthermore, the environment-sensitive pSI provides an adaptive modeling approach for the estimation and prediction of forest site productivity under environmental changes.

Appendix

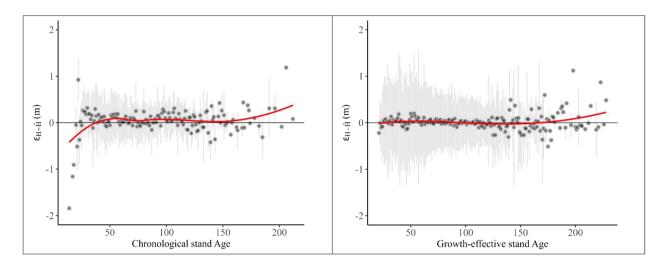


Fig. 11 Residuals of the chronological age based height site index model (left) and the growth-effective age based height site index model (right) plotted versus stand age. Dots: mean, bars: 1.96 standard deviation, lines: loess smoothing line

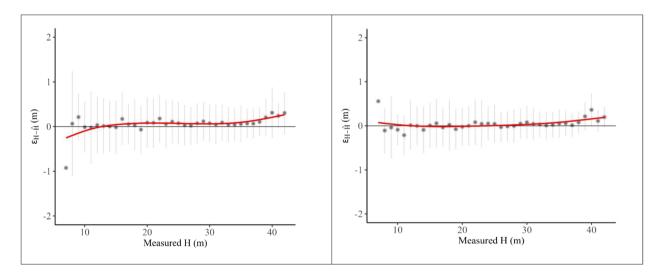


Fig. 12 Residuals of the chronological age based height site index model (left) and the growth-effective age based height site index model (right) plotted versus measured stand height. Dots: mean, bars: 1.96 standard deviation, lines: loess smoothing line

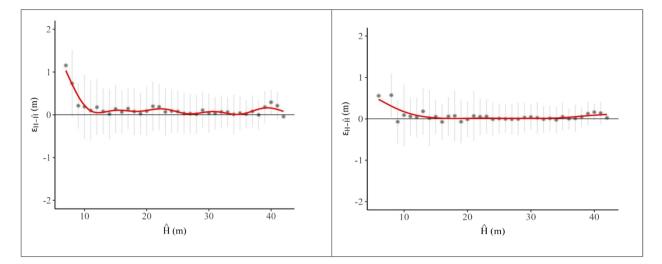


Fig. 13 Residuals of the chronological age based height site index model (left) and the growth-effective age based height site index model (right) plotted versus predicted stand height. Dots: mean, bars: 1.96 standard deviation, lines: loess smoothing line

Table 5 Evaluation of the contributions of site index (<i>SI</i>) and periodic site index (<i>pSI</i>) in the periodic mean annual stand dominant height
increment model (ΔH , Eq. 10) and in the volume increment model (ΔV , Eq. 11) using (i) the existing SI model of Assmann and Franz
(1965) and (ii) the new SI model (Eq. 1) by splitting the data set into training (70%) and validation data (30%)

Sources	Eq.	SI	Training data			Validation data				
			R ²	RMSE	AIC	n	R^2	RMSE	AIC	n
Existing <i>SI</i> model	Δ <i>H</i> (Eq. 10)	SI					0.743	0.097	-9964	5435
		pSI					0.793	0.087	-11,138	5435
	ΔV (Eq. 11)	SI					0.428	5.022	33,002	5435
		pSI					0.471	4.829	32,578	5435
New SI model	ΔH (Eq. 10)	SI	0.742	0.097	-6936	3804	0.744	0.096	-3006	1631
		pSI	0.812	0.083	-8135	3804	0.855	0.072	-3936	1631
	ΔV (Eq. 11)	SI	0.442	4.917	22,932	3804	0.406	5.232	10,046	1631
		pSI	0.447	4.896	22,899	3804	0.433	5.112	9970	1631

R² Coefficient of determination, RMSE Root mean squared error, AIC Akaike information criterion, n Degrees of freedom

Authors' contributions

Conceptualization: Chaofang Yue, Hans-Peter Kahle; methodology: Chaofang Yue, Hans-Peter Kahle; formal analysis and investigation: Chaofang Yue; writing—original draft preparation: Chaofang Yue, Hans-Peter Kahle, Ulrich Kohnle; writing—review and editing: Chaofang Yue, Hans-Peter Kahle, Joachim Klädtke, Ulrich Kohnle; funding acquisition: Joachim Klädtke, Ulrich Kohnle; resources: Ulrich Kohnle, Joachim Klädtke. The authors read and approved the final manuscript.

Funding

Open Access funding enabled and organized by Projekt DEAL. The study was funded by a grant from Fachagentur Nachwachsende Rohstoffe e.V. (FNR, No. 2221NR008A).

Availability of data and materials

The datasets generated during and/or analyzed during the current study are not publicly available but are available from the corresponding author on reasonable request.

Declarations

Ethics approval and consent to participate Not applicable.

Consent for publication

All authors gave their informed consent to this publication and its content.

Competing interests

The authors declare that they have no conflict of interest.

Author details

¹Institute of Forest Research Baden-Württemberg, Freiburg, Germany. ²Institute of Forest Sciences, Chair of Forest Growth and Dendroecology, University of Freiburg, Freiburg, Germany.

Received: 6 February 2024 Accepted: 19 August 2024 Published online: 17 September 2024

References

- Abetz P, Klädtke J (2002) The target tree management system. Forstwiss Centralbl 121:73–82
- Arias-Rodil M, Crecente-Campo F, Barrio-Anta M, Diéguez-Aranda U (2015) Evaluation of age-independent methods of estimating site index and predicting height growth: a case study for maritime pine in Asturias (NW Spain). Eur J For Res 134:223–233. https://doi.org/10.1007/ s10342-014-0845-z
- Assmann E (1970) The principles of forest yield study. Studies in the organic production, structure, increment, and yield of forest stands. Pergamon Press, Oxford, p 506
- Assmann E, Franz F (1965) Vorläufige Fichten-Ertragstafel Für Bayern. Forstw Cbl 84(1):13–43
- Avitabile V, Pilli R, Camia A (2020) The biomass of European forests an integrated assessment of forest biomass maps, field plots and national statistics. Publications Office of the European Union, Luxembourg
- Bailey RL, Clutter JL (1974) Base-age invariant polymorphic site curves. For Sci 20:155–159
- Bontemps JD, Bouriaud O (2014) Predictive approaches to forest site productivity: recent trends, challenges and future perspectives. Forestry 87:109–128. https://doi.org/10.1093/forestry/cpt034
- Bunn A, Korpela M, Biondi F, Campelo F, Mérian P, Qeadan F, Zang C (2023) dplR: dendrochronology program library in R. R package version 1.7.6. https://CRAN.R-project.org/package/dplR
- Burkhart HE, Tomé M (2012) Modeling forest trees and stands. Springer, Netherlands
- Carmean WH (1972) Site index curves for upland oaks in the central states. For Sci 18(2):109–120

- Cieszewski CJ, Bailey RL (2000) Generalized algebraic difference approach: theory based derivation of dynamic site equations with polymorphism and variable asymptotes. For Sci 46:161–26
- Daniel TW, Helms JA, Baker FS (1979) Principles of silviculture. McGraw-Hill, New York, p 500
- Etzold S, Ferretti M, Reinds GJ, Solberg S, Gessler A, Waldner P, Schaub M et al (2020) Nitrogen deposition is the most important environmental driver of growth of pure, even-aged and managed European forests. For Ecol Manag 458:117762
- Fang Z, Bailey RL (2001) Nonlinear mixed effects modeling for slash pine dominant height growth following intensive silvicultural treatments. For Sci 47:287–300
- Fu L, Lei X, Sharma RP, Li H, Zhu G, Hong L, You L, Duan G, Guo H, Lei Y, Li Y, Tang S (2018) Comparing height–age and height–diameter modelling approaches for estimating site productivity of natural uneven-aged forests. Forestry 91:419–433. https://doi.org/10.1093/forestry/cpx049
- García O (2011) Dynamical implications of the variability representation in siteindex modelling. Eur J For Res 130(4):671–675
- Goelz JCG, Burk TE (1998) Long-term trends in height growth of jack pine in North Central Ontario. For Sci 44(1):158–164. https://doi.org/10.1093/ forestscience/44.1.158
- Gregoire TG, Schabenberger O, Barrett JP (1995) Linear modelling of irregularly spaced, unbalanced, longitudinal data from permanent-plot measurements. Can J For Res 25:137–156
- Hann DW, Ritchie MW (1988) Height growth rate of Douglas fir: a comparison of model forms. For Sci 34(1):165–175. https://doi.org/10.1093/forestscie nce/34.1.165
- Hann DW, Hanus ML (2004) Evaluation of nonspatial approaches and equation forms used to predict tree crown recession. Can J For Res 34:1993–2003. https://doi.org/10.1139/x04-076
- Hawryło R, Socha R, Wężyk P, Ochał W, Krawczyk W, Miszczyszyn J, Tymińska-Czabańska L (2024) How to adequately determine the top height of forest stands based on airborne laser scanning point clouds? For Ecol Manage 551:121528. https://doi.org/10.1016/j.foreco.2023.121528
- Henttonen HM, Nöjd P, Mäkinen H (2024) Environment-induced growth changes in forests of Finland revisited - a follow-up using an extended data set from the 1960s to the 2020s. For Ecol Manag 551:121515
- Huang S, Titus S (1993) An index of site productivity for uneven-aged or mixedspecies stands. Can J For Res 23:558–562. https://doi.org/10.1139/x93-074
- Kahle HP, Spiecker H, Unseld R, Pérez Martínez PJ, Prietzel J, Mellert KH (2008) Temporal trends and spatial patterns of height growth changes in relation to changes in air temperature and precipitation, and in relation to levels of foliar nitrogen and nitrogen deposition. In: Kahle HP, Karjalainen T, Schuck A, Ågren G, Kellomäki S, Mellert K et al (eds) Causes and consequences of forest growth trends in Europe - results of the recognition project. Brill, Leiden, pp 127–167. European Forest Institute Research Report 21
- Kangas A, Maltamo M (eds) (2006) Forest inventory methodology and applications. Springer, Dordrecht, p 362
- Kohnle U, Albrecht A, Lenk E, Ohnemus K, Yue C (2014) Growth trends driven by environmental factors extracted from long term experimental data in southwest Germany. Allg Forst Jagdztg 185:97–117
- Leary RA (1985) Interaction theory in forest ecology and management. Springer Netherlands, Dordrecht. http://link.springer.com/10.1007/ 978-94-009-5151-8
- Mäkinen H, Isomäki A (2004) Thinning intensity and growth of Norway spruce stands in Finland. Forestry 77:349–364. https://doi.org/10.1093/forestry/ 77.4.349
- Mäkinen H, Hynynen J, Isomäki A (2005) Effect of intensive management on wood production of Scots pine stands in southern Finland. For Ecol Manag 215:37–50. https://doi.org/10.1016/j.foreco.2005.03.069
- Magnussen S, Penner M (1996) Recovering time trends in dominant height from stem analysis. Can J For Res 26:9–22
- Martin-Benito D, Gea-Izquierdo G, del Rio M, Canellas I (2008) Long term trends in dominant-height growth of black pine using dynamic models. For Ecol Manag 256:1230–1238
- Molina-Valero JA, Diéguez-Aranda U, Álvarez-González JG, Castedo-Dorado F, Pérez-Cruzado C (2019) Assessing site form as an indicator of site quality in evenaged *Pinus radiata* D. Don stands in north-western Spain. Ann For Sci 76:113. https://doi.org/10.1007/s13595-019-0904-1

- Moreno-Fernández D, Álvarez-González JG, Rodríguez-Soalleiro R, Pasalodos-Tato M, Cañellas I, Montes F, Díaz-Varela E, Sánchez-González M, Crecente-Campo F, Álvarez-Álvarez P, Barrio-Anta M, Pérez-Cruzado C (2018) National-scale assessment of forest site productivity in Spain. For Ecol Manag 417:197–207. https://doi.org/10.1016/j.foreco.2018.03.016
- Mosteller F, Tukey JW (1977) Data analysis and regression: a second course in statistics. Addison-Wesley, Reading. ISBN-13: 978-0201048544
- Nigh GD (1996) Growth intercept models for species without distinct annual branch whorls: western hemlock. Can J For Res 26(8):1407–1415
- Noordermeer L, Gobakken T, Nasset E, Bollandsas OM (2020) Predicting and mapping site index in operational forest inventories using bitemporal airborne laser scanner data. For Ecol Manag 457:117768. https://doi.org/ 10.1016/j.foreco.2019.117768
- Nothdurft A, Wolf T, Ringeler A, Böhner J, Saborowski J (2012) Spatio-temporal prediction of site index based on forest inventories and climate change scenarios. For Ecol Manag 279:97–111
- Ols C, Hervé J-C, Bontemps J-D (2020) Recent growth trends of conifers across Western Europe are controlled by thermal and water constraints and favored by forest heterogeneity. Sci Total Environ 742:140453
- Pinheiro JC, Bates DM (2000) Mixed-effects models in S and S-PLUS. Springer, New York. ISBN 0-387-98957-9
- Pretzsch H (2009) Forest dynamics, growth, and yield. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-540-88307-4_1
- Pretzsch H, Biber P, Schütze G, Uhl E, Rötzer T (2014) Forest stand growth dynamics in Central Europe have accelerated since 1870. Nat Commun 5:4967
- Pretzsch H, del Río M, Arcangeli C, Bielak K, Dudzinska M, Forrester DI, Klädtke J et al (2023) Forest growth in Europe shows diverging large regional trends. Sci Rep 13(1):15373
- R Core Team (2023) R: a language and environment for statistical computing. R Foundation for Statistical Computing, Vienna. https://www.R-project.org/
- Raulier F, Lambert MC, Pothier D, Ung CH (2003) Impact of dominant tree dynamics on site index curves. For Ecol Manag 184:65–78. https://doi. org/10.1016/S0378-1127(03)00149-X
- Reineke LH (1933) Perfecting a stand-density index for even-aged forests. J Agr Res 46:627–663
- Riedel T, Hennig P, Kroiher F, Polley H, Schmitz F, Schwitzgebel F (2017) Die dritte Bundeswaldinventur (BWI 2012). Inventur- und Auswertungsmethoden. Bundesforschungsinstitut für Ländliche Räume, Wald und Fischerei, Thünen–Institut für Waldökosysteme, Eberswalde, p 124
- Riofrío J, White JC, Tompalski P, Coops NC, Wulder MA (2023) Modelling height growth of temperate mixedwood forests using an age-independent approach and multi-temporal airborne laser scanning data. For Ecol Manag 543:121137. https://doi.org/10.1016/j.foreco.2023.121137
- Skovsgaard JP, Vanclay JK (2008) Forest site productivity: a review of the evolution of dendrometric concepts for even-aged stands. Forestry 81:13–31. https://doi.org/10.1093/forestry/cpm041
- Skovsgaard JP, Vanclay JK (2013) Forest site productivity: a review of spatial and temporal variability in natural site conditions. Forestry 86:305–315. https://doi.org/10.1093/forestry/cpt010
- Sloboda B (1971) Zur Darstellung von Wachstumsprozessen mit Hilfe von Differentialgleichungen erster Ordnung, Mitteilungen der Forstlichen Versuchs- und Forschungsanstalt Baden-Württemberg, p 32
- Socha J, Pierzchalski M, Ciesielski M (2017) Modelling top height growth and site index using repeated laser scanning data. For Ecol Manag 406(15):307–317. https://doi.org/10.1016/j.foreco.2017.09.039
- Spiecker H, Mielikäinen K, Köhl M, Skovsgaard JP (eds) (1996) Growth trends in European forests - studies from 12 countries. Springer-Verlag, Berlin. (European Forest Institute Research Report 5)
- Tompalski P, White JC, Coops NC, Wulder MA (2019) Demonstrating the transferability of forest inventory attribute models derived using airborne laser scanning data. Remote Sens Environ 227:110–24. https://doi.org/10. 1016/j.rse.2019.04.006
- Tompalski P, Coops NC, White JC, Goodbody TRH, Hennigar CR, Wulder MA, Socha J, Woods ME (2021) Estimating changes in forest attributes and enhancing growth projections: a review of existing approaches and

future directions using airborne 3D point cloud data. Curr For Rep 7:1–24. https://doi.org/10.1007/s40725-021-00135-w

- Tomppo E, Gschwantner T, Lawrence M, McRoberts RE (2010) National forest inventories: pathways for common reporting. Springer, Heidelberg. ISBN: 978-9-04-813232-4
- Vanclay JK (1992) Assessing site productivity in tropical moist forests: a review. For Ecol Manag 54:257–287. https://doi.org/10.1016/0378-1127(92) 90017-4
- Vanclay JK, Henry NB (1988) Assessing site productivity of indigenous cypress pine forest in southern Queensland. Commonw For Rev 67(1):53–64
- Wakeley PC, Marrero J (1958) Five-year intercept as site index in southern pine plantations. J For 56(5):332–336. https://doi.org/10.1093/jof/56.5.332
- Wang GG (1998) Is height of dominant trees at a reference diameter an adequate measure of site quality? For Ecol Manag 112:49–54. https://doi. org/10.1016/S0378-1127(98)00315-6
- Wang Y, Huang S, Yang RC, Tang S (2004) Error-invariable method to estimate parameters for reciprocal base-age invariant site index models. Can J For Res 34:1929–1937
- Wang Y, LeMay V, Baker TG (2007) Modelling and prediction of dominant height and site index of Eucalyptus globulus plantations using a nonlinear mixed-effects model approach. Can J For Res 37:1390–1403
- Weiskittel AR, Hann DW, Kershaw JA, Vanclay JK (2011) Forest growth and yield modeling. Wiley-Blackwell, Oxford, p 344
- Yue C, Mäkinen H, Klädtke J, Kohnle U (2014) An approach to assessing site index changes of Norway spruce based on spatially and temporally disjunct measurement series. For Ecol Manag 323:10–19. https://doi.org/ 10.1016/j.foreco.2014.03.031
- Yue C, Kahle HP, von Wilpert K, Kohnle U (2016) A dynamic environmentsensitive site index model for the prediction of site productivity potential under climate change. Ecol- Model 337:48–62. https://doi.org/10.1016/j. ecolmodel.2016.06.005

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.